THE YOUTH'S ASSISTANT IN THEORETICK AND PRACTICAL ARITHMETICK.

DESIGNED FOR THE

Use of Schools in the United States.

BY ZADOCK THOMPSON, A. M.

Author of the Gazetteer of the State of Vermont.

SECOND EDITION,

WITH CORRECTIONS AND ADDITIONS.

WOODSTOCK, VT.

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1826.

DISTRICT OF VERMONT, ss.

L. S.

BE IT REMEMBERED, that on the eighth day of September, in the fifty first year of the Independence of the United States of America, ZADOCK THOMPSON, of the said District, hath deposited in this office the title of a Book, the right whereof he claims as Author, in the words following, to wit:

"The Youth's Assistant in Theoretick and Practical Arithmetick. Designed for the use of Schools in the United States. By Zadock Thompson, A.M. Author of the Gazetteer of the State of Vermont. Second edition, with corrections and additions."

In conformity to the act of the Congress of the United States, entitled, "an act for the encouragement of learning, by securing the copies of maps, charts and books to the authors and proprietors of such copies during the times therein mentioned."

JESSE GOVE,
Clerk of the District of Vermont.

A true copy of record, examined and sealed by me,
J. GOVE, *Clk.*

PREFACE

TO THE SECOND EDITION.

In presenting a new edition of the Youth's Assistant, the Author begs leave to tender his grateful acknowledgment for the liberal patronage his work has already received from an enlightened publick. By his own observation and experience in teaching, he had become convinced that the Arithmeticks hitherto published, to say nothing of the want of order in the arrangement of the rules, were, in general, either very deficient in demonstration, or too full and expensive for the use of common schools. He, therefore, commenced this work with the view of uniting the simplicity of the one with the demonstration of the other, hoping at the same time, by an orderly distribution of the rules into classes, according to their similitude and dependence, and by a rejection of every thing which is not of real use to the scholar, to afford him more valuable matter than is to be found in any one of our common school Arithmeticks, and at less expense. How far he has succeeded, is not for him to say. But from the rapid sale of the preceding edition, he flatters himself that his attempt was not regarded as altogether fruitless. In the present edition, very considerable additions and improvements have been made. Besides the correction of many typographical errors, the number of practical examples has been much increased, several new notes have been added, the part on Mental Arithmetick has been enlarged, and a short but comprehensive system of Book-Keeping by Single Entry has been introduced. New rules have also been substituted, in a few cases, in place of those given in the former edition, where it was thought the student would be benefited by the change, particularly in Duodecimals and Compound Proportion. As it was the Author's design to combine the excellencies of the Arithmeticks now in use, in a lucid and perspicuous manner, he has not aimed at originality, but has selected freely from the works which came to his hand, such materials as he deemed suited to his purpose.

Grateful to those instructers, whose suggestions have led to many of the improvements in the present edition, the Author now submits it to their inspection, hoping it may be found still worthy of that approbation which they have so freely expressed, and that it may, in some measure, subserve the advancement of youth in the important science of numbers.

Burlington, Oct. 12, 1826.

MENTAL ARITHMETICK.

INTRODUCTION.

THAT frequent exercise in mental computations have a happy influence upon the mind, by inducing habits of attention, by strengthening the memory, and by producing a promptness of recollection, is, at present, universally admitted. And, that exercises of this nature should be more extensively introduced into our primary schools, is acknowledged and even urged by many of our most experienced and successful instructers. The success, which has, in most cases, attended the introduction of Mental Arithmetick, would doubtless appear incredible to those unaccustomed to it. But experience has shown, that children may, in this way, be made acquainted with the first principles of Arithmetick, at as early an age as they can be taught the Alphabet and its most simple combinations. Classes of children under nine years of age, by the force of memory and the aid of a few plain rules, without the assistance of pen or pencil, have been taught to multiply seven or eight figures by an equal number, enumerate and announce accurately the product, amounting to quintillions; and then extract the square root of this large product, and state the root and remainder, without varying a figure from the truth.

In the ordinary course of instruction, Arithmetick has been studied only by the boys; and it has usually been deferred by them to the last portion of their attendance at school. The consequence has been, that few have become familiar with its first principles, before they have been obliged to quit school, and enter upon the business of life. Commencing the study of Arithmetick at this advanced period, the scholar is sensible he has but little time to devote to it, but, being determined *to cipher through his book*, he applies himself with diligence, yet he hurries on from rule to rule with such rapidity, that he learns nothing as he ought. He may indeed *reach the end*, and thus accomplish his principal purpose; but, of what he has gone over, scarcely a trace remains upon his mind. He has not even made himself thoroughly acquainted with any of the elements of the science; nor has he made himself so familiar with the rules, as to derive from them any considerable practical advantage. It is asserted with confidence, that children, after having learned to talk, cannot too soon be made acquainted with numbers, and exercised in mental computations. But care should be taken, that these exercises be adapted to the age and capacity of the child —that the questions proposed, be such as the child can fully comprehend. And as very young children are scarcely capable of the exercise of abstraction, the instructer will derive much advantage from the employment of sensible objects. The child will find no difficulty in conceiving sensible objects, that are before him, to represent such as are absent, even at an age when they could form no conception of abstract numbers. After making the child acquainted with the nature of the several operations by means of these objects, he should be directed to perform the same by the force of memory, and he will, in this way, soon become familiar with fundamental principles of Arithmetic, and his judgment, as it becomes matured by age, will direct him in the application of these principles to practical purposes. He will at the same time be acquiring habits of attention, and a promptness of computation, which will be of inestimable value to him in after life. All this may be done as an amusement, and a relaxation

to the young mind, without interrupting, in the least, its other pursuits; and thus may every boy and girl, of ordinary capacity, be made more thoroughly acquainted with the elements of Arithmetick, before they arrive at the age of ten years, than most of our scholars are on leaving school, after having plodded through the whole Arithmetick in the ordinary way. A knowledge of Arithmetick, at the present day, is scarcely less necessary to the female sex than to our own, and if the course be adopted, which is here recommended, it is believed, they will not be found less capable of proficiency in this science It is hoped that our teachers, both male and female, will take this subject into consideration, and use their exertions to bring about a reformation so desirable in the course of arithmetical instruction.

ADDITION.

1. I have two cents in one hand, and one in the other; how many have I in both?

2. How many fingers have you on one hand? 3. How many on both?

4. If you count your thumbs with your fingers; how many will it make?

5. George has three apples in one pocket and two in the other; how many has he in both?

6. How many are three and two?

7. Henry has four cents and Geo. two; how many have both?

8. David gave three cents for a lemon and five for an orange; how many did he give for both?

9. Three and five are how many?

10. Peter had six cherries, and Dick gave him four more; how many had he then?

11. John had seven nuts and Peter gave him two more; how many had he then?

12. A man bought a barrel of flour for seven dollars, and a barrel of soap for four dollars; how much did they both cost?

13. A man has six cows at one barn, and eight at another; how many has he at both?

14. Eight and six are how many?

15. A person bought a hundred weight of sugar for ten dollars, and a barrel of flour for seven dollars; how much did he give for both?

16. A man travelled four miles the first hour, three the next, and two the next; how far did he travel in the three hours?

17. If I give nine dollars for three sheep, and ten dollars for a cow, how much do I give for the cow and sheep?

18. Eight and four are how many?

19. Nine and five are how many?

20. Seven and seven are how many?

21. Seven & eight are how many?

22. Nine & eight are how many?

23. Nine & ten are how many?

24. Nine & nine are how many?

25. Eleven & nine are how many?

26. Twelve & nine are how many?

27. A boy gave to another boy six peaches, to another four, and had three left; how many had he at first?

28. A boy bought a slate for 22cts. a pencil for three, and a sponge for six; how much did they all cost?

29. A man gave seven dollars for a sleigh, gave six dollars for ironing it, and four dollars for painting it; what did the whole cost?

30. A drover bought twenty-three sheep of one man, seven of another, and five of another; how many did he buy of the three?

31. A lady bought some pins for eleven cents, a comb for twenty-five, and some tape for eight cents; what did they all cost?

32. Nine and eight and six are how many?

33. Five and three and eleven are how many?

34. Seven and four and twelve are how many?

35. Thirty-five and six and four are how many?

36. Forty-seven and three and seven are how many?

37. A man bought a cow for seventeen dollars, a hog for five dollars,

and three sheep for six dollars; what did they all cost?

38. From Burlington to Montpelier it is thirty-eight miles, and from Montpelier to Woodstock it is forty-seven miles; how far is it from Burlington to Woodstock?

39. A man bought a horse for forty eight dollars, a yoke of steers for twenty-three dollars and a half, and a cow for fourteen dollars and a half; how much did he give for the three?

40. How many are nineteen, and nine, and twenty-nine?

41. A boy paid ten cents for a card of gingerbread, six cents for a pint of plumbs, lost twenty-eight cents at play, and had eleven cents left; how many had he at first?

42. How many are seventeen and seven and seventy-six?

SUBTRACTION.

1. David had six plums, and gave two of them to George; how many had he left?

2. A boy had eight cents and lost three of them; how many had he left?

3. A man bought a barrel of flour for eight dollars, and sold it again for twelve dollars; how much did he gain by the trade?

4. A person bought nineteen lbs. of rice, and having lost a part of it, found he had nine pounds left; how much did he lose?

5. A boy having twenty cents, bought one quart of plums for six cents, and a pound of figs for ten cents; how many cents had he left?

6. A man bought a cow for sixteen dollars, and sold it again for twelve dollars; how much did he lose?

7. Seven less three are how many?

8. Eight less three are how many?

9. Eleven less four are how many?

10. Twenty-one less four are how many?

11. Thirty less six are how many?

12. Six and ten less four are how many?

13 Nine and fifteen less eight are how many?

14. A lady bought a comb for thirty-three cents, some tape for eight cents, and some needles for six cts. She gave fifty cents; how much change must she receive?

15 Peter had twelve cents, and John gave him ten more, with which he bought eleven cents worth of cake; how many cents had he left?

16 Twenty-one less nine are how many?

17. Twenty-seven less eleven are how many?

18. A man owed seventy-five dollars, of which he paid at one time fifteen, and at another twenty-five dollars; how much remains to be paid?

19. Twenty-five less eight and six are how many?

20. A person bought a horse for sixty dollars, a saddle for twenty dollars, and a bridle for two dollars and a half, and sold them all together for eighty-six dollars; did he gain or lose? and how much?

21. Twenty-one and eight and seven less seventeen are how many?

22. Thirty and forty less twenty and twenty-five are how many?

23. A lady bought two yards of calico for sixty-two cents, a yard of ribbon for twenty-one cents, and two skeins of silk for eight cents, and gave a dollar bill; how much should she receive back?

24. A barrel containing thirty-two gallons of cider, sprung a leak, and nine gallons run out; how much was there left?

25. A man sold a drover seven sheep for twelve dollars, a yoke of oxen for sixty-eight dollars, two cows for twenty-six dollars, and received in payment one hundred dollars; how much remains his due?

26. If I buy a horse for seventy dollars, and a saddle for nineteen dollars, and sell them both for ninety-five dollars, do I lose or gain, and how much?

MULTIPLICATION.

1. What cost two apples at one cent a piece?

2. What cost two lemons at two cents a piece?

3. What cost four yards of tape at two cents a yard?

4. What cost three barrels of cider at three dollars a barrel?

5. At four cents a piece what will three oranges cost?

6. At four cents a yard what will four yards of ribbon cost?

7. If a man travel three miles an hour, how far will he travel in six hours?

8. What will eight yards of cloth cost at four dollars a yard?

9. What will six pounds of raisins cost at nine cents a pound?

10. What will seven yards of shirting cost at three shillings a yard?

11. What will sixteen yards of shirting cost at ten cents a yard?

12. If four bushels of wheat make a barrel of flour; how many bushels will it take to make eight barrels?

Multiplication Table.

How many are

Two times two?	Four times five?	Six times twelve?
Two times three?	Four times six?	Seven times seven?
Two times four?	Four times seven?	Seven times eight?
Two times five?	Four times eight?	Seven times nine?
Two times six?	Four times nine?	Seven times ten?
Two times seven?	Four times ten?	Seven times eleven?
Two times eight?	Four times eleven?	Seven times twelve?
Two times nine?	Four times twelve?	Eight times eight?
Two times ten?	Five times five?	Eight times nine?
Two times eleven?	Five times six?	Eight times ten?
Two times twelve?	Five times seven?	Eight times eleven?
Three times three?	Five times eight?	Eight times twelve?
Three times four?	Five times nine?	Nine times nine?
Three times five?	Five times ten?	Nine times ten?
Three times six?	Five times eleven?	Nine times eleven?
Three times seven?	Five times twelve?	Nine times twelve?
Three times eight?	Six times six?	Ten times ten?
Three times nine?	Six times seven?	Ten times eleven?
Three times ten?	Six times eight?	Ten times twelve?
Three times eleven?	Six times nine?	Eleven times eleven?
Three times twelve?	Six times ten?	Eleven times twelve?
Four times four?	Six times eleven?	Twelve times twelve?

13. What will twelve pounds of butter cost, at twelve and a half cents a pound?

14. If a person earn five dollars a week, how much does he earn in twelve weeks?

15. If a person earn seven shillings a day, how much does he earn in nine days?

16. At twelve cents a pound, what will eleven pounds of sugar cost?

17. Peter has fifteen cents, and John has three times as many; how many has John?

18. A certain room has four windows, each containing eighteen panes of glass; how many panes are there in the whole?

19. A ream of paper contains twenty quires, of twenty-four sheets each; how many sheets in a ream?

20. What will nineteen bushels of potatoes cost, at eighteen cents a bushel?

21. What will twenty-one barrels of cider cost, at seventy-five cents a barrel?

22. What will fifty sheep cost, at one dollar twenty cents a piece?

DIVISION.

1. If two apples cost four cents, how much is that a piece?

2. If three lemons cost nine cents, how much is that a piece?

3. A lad had twelve plums, which he divided equally among six boys; how many did each have?

4. If you divide twenty dollars equally among four men; how much will each have?

5. In fifteen how many times five?

6. In twenty-one how many times seven?

7. In sixteen how many times four?

8. In thirty how many times five?

9. In twenty-four how many times eight?

10. In eighteen how many times two?

11. If a quire of paper cost twelve cents, how much is that a sheet?

12. If five lemons cost thirty cents, how much is that a piece?

13. If fifty-four cherries be divided among six girls, how many will they have a piece?

14. At twelve cents a dozen, how much will half a dozen apples cost?

15 If five pounds of sugar cost fifty cents, what is that per pound?

16. If twenty yards of cloth cost six dollars, what is that a yard?

17. In a certain cornfield are eight rows forty hills long; how many hills are there?

18. A certain cornfield contains three hundred and twenty hills, and the rows are forty hills long; how many rows are there?

19. In five hundred, how many times twenty?

20. If a bushel of wheat cost eighty cents, how much is that a quart; there being four pecks in a bushel, and eight quarts in a peck?

21. In two dozen and a half, how many half dozen?

22. In a certain village are two hundred and eighty-five persons, and the average number in each family is five; how many families are there?

FRACTIONS.

1. How much is one half and one fourth?

2. How much is one half and three fourths?

3. How much is five thirds and four thirds?

4. If you take one fourth from one half, what remains?

5. One third from two, what remains?

6. One third and two sixths from one, what remains?

7. One fourth and one half from two, what remains?

8. If an apple be divided into seven equal parts, and you give away five of them, how would you express the value of the parts disposed of?

9. How the parts remaining?

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Explanation of Characters.

$=$ Equality — is expressed by two horizontal marks; thus, 100 cts.$=$1 dollar, signifies that 100 cents are equal to 1 dollar.

$+$ Addition — is expressed by a cross formed by one horizontal, and one perpendicular mark. Thus $4+9=9$ signifies that 5 added to 4, is equal to 9.

$-$ Subtraction — is expressed by one horizontal mark between the numbers. Thus $7-4=3$ signifies that 4 taken from 7, the remainder is equal to 3.

$\times$ Multiplication — is expressed by a cross formed by two oblique marks. Thus $5\times3=15$ signifies that 5 multiplied by 3, the product is equal to 15.

$\div$ or) (Division — is expressed by a horizontal line with a dot on each side, or by a reversed parenthesis. Thus $6\div2=3$, or 2)6(3 signifies that 6 divided by 2, the quotient is equal to 3.

: :: : Proportion — is expressed by four colons. Thus 2 : 6 :: 8 : 24 signifies that 2 has the same proportion to 6, that 8 has to 24. In arithmetical proportion two of the colons are placed horizontally; thus 2 ·· 4 :: 6 ·· 8.

$\overline{}|^2$ signifies the second power, or square of the number over which it is placed. Thus $\overline{6}|^2$ denotes the square of 6, or $6\times6=36$.

$\overline{}|^3$ signifies the third power, or cube of the number. Thus $\overline{6}|^3$ denotes the cube of 6, or $6\times6\times6=216$.

$\overline{}|^{\frac{1}{2}}$ signifies the square root. Thus $\overline{36}|^{\frac{1}{2}}$ denotes the square root of 36, or 6.

$\overline{}|^{\frac{1}{3}}$ signifies the cube root.

$8-\overline{4+2}=2$, shows that the sum of 2 and 4, subtracted from 8, is equal to 2. The line over the 4 and 2 is called a *vinculum*.

INTRODUCTION.

ARITHMETICK.

Arithmetick is the science of numbers, and is of two kinds, *theoretick* and *practical.*

Theoretick Arithmetick treats of the nature of numbers, and shews the foundation of the rules for practical operations.*

Practical Arithmetick is the method of applying these rules to the solution of questions and the transaction of business.

In entering upon the study of Arithmetick, the first objects which demand the student's attention are Notation and Numeration. With these he should endeavour to become familiar, as a knowledge of them is indispensable at every step of his future progress.

NOTATION.

Notation is the method of writing, or expressing, any proposed number in characters or figures.

There are two methods of expressing numbers, the *Roman* and the *Arabick.*

The *Roman method* is by letters of the alphabet. It is at present but little used, except in numbering chapters, sections, and the like.

The *Arabick method* is by characters, and is the one in general use.

The Arabick characters* or figures are the ten following; 1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, and 0 cipher.† By repeating and varying the position of these ten characters, all numbers whatever may be expressed. The nine first are called significant figures, or digits, because they invariably point out, or express some particular number. The cipher standing alone has no signification; but placed at the right hand of a significant figure, it increases the value of that figure in a ten fold proportion. Thus 4 standing alone is four; annex a cipher (40,) and it is forty; another cipher (400,) and it is four hundred. Hence figures have a local as well as simple value; and the local value depends on the distance of the figure from the right hand, or place of units, each removal to the left increasing the value of the figure in a ten fold proportion. And the effect is the same, whether the places on the right be filled with ciphers or significant figures. But this will appear more clearly from what follows.

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* The Theory of Arithmetick contained in this treatise will be mostly found in the Notes.

† These characters were introduced into Europe, from Arabia, by the Saracens about A.D. 991. The letters of the alphabet were used previous to this time. The following table exhibits a comparison of the two methods of notation.

1 I.	6 VI.	11 XI.	16 XVI.	10 X.	60 LX.	100 C.	600 DC.
2 II.	7 VII.	12 XII.	17 XVII.	20 XX.	70 LXX.	200 CC.	700 DCC.
3 III.	8 VIII.	13 XIII.	18 XVIII.	30 XXX.	80 LXXX.	300 CCC.	800 DCCC.
4 IV.	9 IX.	14 XIV.	19 XIX.	40 XL.	90 XC.	400 CCCC.	900 DCCCC.
5 V.	10 X.	15 XV.	20 XX.	50 L.	100 C.	500 D or IƆ.	1000 M.

NUMERATION.

Numeration teaches how to read any proposed number expressed in figures. We have already observed that figures have a local as well as simple value. The method of determining these values may be learnt from the following

Numeration Table.

Trillions.	Hun. of Thou. of Bill.	Tens of Thou. of Bill.	Thousands of Bill.	Hundreds of Billions.	Tens of Billions.	Billions.	Hun. of Thou. of Mill.	Tens of Thou. of Mill.	Thousands of Mill.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thous.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
6	4	6	8	7	6	4	2	1	3	8	5	1	2	6	7	9	8	7

The words at the head of the Table show the names of the several places against which they stand, or the local value of the figures occupying those places. These words should, in the first place, be perfectly committed to memory, as they are applicable to any sum or number. This being done, to enumerate any sum, observe the following RULE.—*To the simple value of each figure, join the name of its place, beginning at the left, and reading towards the right.* Thus 8 being in the ten's place in the table, is eighty, 9 in the hundred's place is nine hundred, 7 in the thousand's place is seven thousand, &c. Hence the whole number would read thus: six trillions, four hundred and sixty-eight thousands, seven hundred and sixty-four billions, two hundred and thirteen thousands eight hundred fifty-one millions, two hundred sixty-seven thousands nine hundred and eighty-seven.

Application.

Write the following numbers in their proper figures. Eight. Nineteen. Eighty. Three hundred and sixty-nine. Five thousand, three hundred and seven. Thirty thousand and fifty-nine. Two billions. Three trillions, six billions, seven millions and one hundred thousands.

Write the following numbers in words: 9, 26, 348, 4090, 84704, 514242, 42357440, 800000, 6000000, and 260400100220160.

For the more easy reading of large numbers, it is customary to divide them into periods and half periods, as in the following Table:

Periods.	Sextil.	Quintil.	Quadril.	Trillions.	Billions.	Millions.	Units.
Half Per.	th. un.	th. un.	th. un.	th. un.	th. un.	th. un.	ext. exu.
Figures.	211.974	321,234	108,642	320,123	458.620	512.345	254,162

Here it will be observed that in enumerating, the same names are repeated in each of the periods, and then the name of the period annexed. Thus the first period is two hundred fifty-four thousand, one hundred and sixty-two *units*; the second, five hundred, twelve thousand, three hundred forty-five *millions*; the third, four hundred, fifty-eight thousand, six hundred and twenty *billions*, &c. Hence a number consisting of twenty, thirty, forty, or more figures, after dividing it into periods, and knowing the name of each, can be enumerated with the same ease as one consisting of six figures only.

ARITHMETICK.

PART I.

FUNDAMENTAL RULES.

The Fundamental Rules of Arithmetick are four, Addition, Subtraction, Multiplication and Division. They are called Principal or Fundamental Rules, because one, or more of them is concerned in all arithmetical operations. Each of these rules is of two kinds, *simple* and *compound.* They are *simple* when the numbers employed are all of one sort, or denomination; and *compound* when the numbers employed are of different denominations.

After having made himself familiar with Notation and Numeration, the scholar's next business is to obtain a thorough knowledge of the four fundamental rules. If these are passed over slightly, he will, in his future progress, meet with continual embarrassment. But if he becomes master of each rule before he proceeds to the next, those difficulties, which would otherwise obstruct his progress, *will entirely vanish, or be surmounted with ease.*

SECTION I.

SIMPLE RULES.

1. Simple Addition.

Simple Addition is the putting together of several numbers of the same denomination, into one whole or total number, called the *sum,* or *amount.* Thus 5, 4 and 3 put together, their sum is 12.

Rule.*—1. Write the numbers to be added under one another with units under units, tens under tens, and so on, beginning at the

* This rule and the method of proof, are founded on the well known axiom, "*the whole is equal to the sum of all its parts.*" The reason of carrying for ten is that ten in any inferior column, is just equal to one in the next superior, as is evident from the nature of Notation. There are several other methods of proving Addition, besides those given above. A very ingenious one is by casting out the *nines*, and as this method, with the proper variations, is applicable to all the fundamental rules, I shall proceed to explain it. The nines are cast out of a sum by beginning at one end of the line of figures, and adding them together, rejecting nine from the sum as often as it occurs. *Example.*—Cast the nines out of 14838. Beginning at the right hand, say 8 to 3 is 11, this being 9 and 2 over, drop the 9 and say 2 to 8 is 10, reject 9 again, and say 1 to 4 is 5, and 1 is 6. 6 then is the excess after casting the 9's out of 14838. Proceed in the

right, and proceeding towards the left, and draw a line below them.

2. Add the right hand column, and if the sum be less than ten, write it below the line at the foot of the column; but if it be ten, or more than ten, write down the excess of tens, and carry as many units as there are tens to the next column, with which proceed as before; and so on till the whole is finished, remembering at the last column to set down the whole amount.

PROOF.—Cut off the upper line of figures, and find the sum of the rest; add this sum to the upper line, and, if their sum be the same as the first amount, or sum total, the work is right; or, which is commonly practised, begin at the top and reckon downwards, and if it be right, this sum will be the same as the first amount.

Addition and Subtraction Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	5	6	7	8	9	10	11	12	13	14
3	5	6	7	8	9	10	11	12	13	14	15
4	6	7	8	9	10	11	12	13	14	15	16
5	7	8	9	10	11	12	13	14	15	16	17
6	8	9	10	11	12	13	14	15	16	17	18
7	9	10	11	12	13	14	15	16	17	18	19
8	10	11	12	13	14	15	16	17	18	19	20
9	11	12	13	14	15	16	17	18	19	20	21
10	12	13	14	15	16	17	18	19	20	21	22

same way with all other numbers. Then to prove Addition by casting out the 9's, observe the following RULE.—Cast the 9's out of each line of figures in the given sum, and write the excesses in a column at the right hand. Add the excesses, and the 9's being cast out of this sum and the sum of the given numbers, if the two excesses are equal, the Addition is supposed to have been correct.

EXAMPLE.

	Excesses
16423	7
21230	8
90418	4
———	–
128071	1

Here the excess of the first line is 7, the second 8, the third 4, and the sum of the excesses is 19. Then casting the 9's out of 19, and also out of the sum 128071, and the excess, in both cases is 1; therefore, the work is supposed to be right. This method of proof is in all cases subject to the inconvenience that a wrong operation may sometimes appear to be right, for if we change the places of any two figures in the sum, the result will still be the same.

This method depends upon a property of the number 9, which belongs to no other number, except 3, namely, that any number divided by 9 leaves the same remainder as the sum of its digits divided by 9. Thus 436 divided by 9, the remainder is 4; the sum of the digits in 436 is (4+3+6=)13 and 13 divided by 9, the remainder is 4 as in the former case, and the same is true universally, as may be analytically demonstrated. It may be proved by Subtraction, thus; Beginning at the left, add again each of the several columns, subtract the sums from the sums at the foot of the columns respectively, and write down the remainders, which must be joined each as so many tens to the sum of the column on the right; if the work be right, there will be no remainder under the last column. See Example above. Here we add the left hand column, and find the sum 12, this taken from 12 leaves 0; then add the next, and find it 7; this from 8 leaves 1: the sum of the next is 10, this from 10 leaves 0; the next is 6; this from 7 leaves 1; the next is 11, this from 11 leaves 0; it is therefore right.

Examples.

1. What is the sum of 6432, 8224, 101 and 81, when added together?

	Thous.	Hund.	Tens.	Units.
	6	4	3	2
	8	2	2	4
		1	0	1
			8	1
Ans. 1	4	8	3	8
	8	4	0	6
Proof. 1	4	8	3	8

Having written the numbers according to the rule, and drawn a line below them, begin with the right hand column and say, 1 to 1 is 2, and 4 is 6, and 2 is 8, which being less than *ten*, write it below the line at the foot of the column. Then proceed to the next column and say, 8 to 0 is 8, and 2 is 10, and 3 is 13; 13 being one 10 and 3 over, write down 3, the excess, and carry 1 to the next column, saying 1 to 1 is 2, and 2 is 4, and 4 is 8, which write down. Then 8 to 6 is 14; this being the last column, write down the whole number by placing 4 the excess of tens, under the column, and 1 the number of tens, at the left hand.

To prove that the operation has been rightly performed, cut off the upper line of figures, and add the three lower lines as already taught, setting their sum, 8406, below a line drawn under the first amount, with each figure directly under the line which produced it; add this last sum to the upper line, and their sum, 14838, being the same as the answer, or amount of all the given numbers, the work is considered to be right.

By careful attention to this rule and its illustration by the preceding example, the student will find but little difficulty in working the examples which follow.

2.	3.	4.	5.
16423	3214	8642	1234567
21230	1032	3124	7654321
90418	3521	4213	1234567
128071 Ans.			
111648			
128071 Proof.			

6.	7.	8.	9.
8192735	123456789	9876987	40004
214268	12345678	7986698	100606
1541320	1234567	4343434	202020
40212	123456	2121212	1219

Application of the Rule.

As arithmetick is usually studied on account of its practical utility in transacting the business of life, it is important that the scholar acquire such a knowledge of each rule as he proceeds, as will enable him readily to apply it in practice, whenever occasion shall require. To aid him in obtaining this knowledge, and to give a scope for the exercise of his judgment, a great variety of such questions will be given under each rule as will be most likely to occur in the transaction of business.

1. What is the amount of 2565 dollars, 796 and 7009 dollars, when added together?

```
        2 5 6 5
        -------
          7 9 6
        7 0 0 9
      ---------
Ans.  1 0 3 7 0
      ---------
        7 8 0 5
      ---------
Proof. 1 0 3 7 0
```

2. A man has 3 fields of grass to cut, one containing 12, another 23, and another 47 acres; how many acres are there in the whole? Ans. 82 acres.

3. Supposing a man has three barns, and at one of them he keeps 6 oxen, 8 cows, and 16 calves, at another, 10 cows and 2 oxen, and at the other, 8 steers and 11 heifers; how many cattle are there in the whole?
Ans. 61.

4. The number of free white males in the United States at the last census, (1820,) was 3,995,-053, the number of free white females, 3,866,657, and the number of every other description, 1,776,289, what was the whole number of inhabitants in the United States at that time?
Ans. 9,637,999.

5. Sir I. Newton was born in the year 1642, and was 85 years old when he died; in what year did he die? Ans. 1727.

6. In a certain town there are 8 schools, the number of scholars in the first is 24, in the second, 32, in the third 28, in the fourth 36, in the fifth 26, in the sixth 27, in the seventh 40, and in the 8th 38; how many scholars in all the schools? Ans. 251.

7. How many days in the 12 calendar months? Ans. 365.

8. I have 100 bushels of wheat worth 125 dollars, 150 bushels of rye worth 90 dollars, and 90 bushels of corn, worth 45 dollars; how much grain have I, and what is it worth? Ans. 340 bushels worth 270 dollars.

9. There are two numbers, the least is 2575, and the difference is 1448, what is the greater?
Ans. 4023.

10. A man killed 4 hogs weighing as follows; one 371, one 510, one 472, and one 396 pounds; what did they all weigh?
Ans. 1749 lbs.

11. A man killed an ox, the meat weighed 642, the hide 105, and the tallow 92 pounds; what did they all weigh?
Ans. 839 lbs.

QUESTIONS.

1. What are the fundamental rules of arithmetick?
2. Why are they called principal or fundamental rules?
3. Of how many kinds is each of these rules?
4. When are they simple? When compound?
5. What is Simple Addition?
6. What is the whole or total number called?
7. How do you write down the numbers to be added?
8. Where do you begin the addition?
9. How is the amount of each column to be set down?
10. What do you observe respecting the sum of the last column?
11. How is Addition proved?
12. Why do you carry for 10 rather than any other number?

2. Simple Subtraction.

Simple Subtraction is the method of taking a less number from a greater of the same denomination, so as to find the difference between them; as, 5 taken from 8, there remains 3, the difference.

The greater of the given numbers is called the *Minuend*, the less, the *Subtrahend*, and the difference, the *Remainder*.

Rule.*

Write the less number under the greater, with units under units, tens under tens, and so on, and draw a line below them. Beginning at the right hand, take each figure of the subtrahend from the figure over it in the minuend, and set the remainder directly below.

If the figure in the lower line be greater than the one over it, suppose 10 to be added to the upper figure, always remembering, in such cases, to carry 1 to the next figure in the lower line, with which proceed as before; and so on till the whole is finished.

Proof.

Add the remainder to the subtrahend, and if their sum be equal to the minuend, the work is right.

Examples.

1. From 6485 subtract 4293.

Minuend.	6 4 8 5
Subtrahend.	4 2 9 3
Remainder.	2 1 9 2
Proof.	6 4 8 5

Having placed the numbers as the rule directs, begin at the right hand, and say, 3 from 5 there remains 2, which write down, and proceed to the next figure, and say, 9 from 8; but 8 being less than 9, you must suppose 10 to be added to 8, making it 18, then say 9 from 18 there remains 9, which write down. Proceed to the next figure, but because you borrowed 10 in the last place, you must carry 1, saying 1 carried to 2 is 3, and 3 from 4 there remains 1, which write down, and proceed again; 4 from 6, there remains 2, which set down, and the work is done.

Proof. To know whether you have performed the operation correctly,

---

* When the figures in the subtrahend are all less than their correspondent figures in the minuend, the sum of the several differences is evidently the true difference between the numbers; for as the sum of the parts is equal to the whole, so is the sum of the differences of the similar parts equal to the difference of the whole. And when the figure in the subtrahend is greater than its correspondent figure in the minuend, borrowing ten, which is the value of a unit in the next higher place, is in fact employing a unit of the next left hand figure of the minuend, before you arrive to it. But as this figure is not actually diminished, the true balance is preserved, by increasing its correspondent figure in the subtrahend by 1. If, when we borrow 10, we diminished the next figure in the minuend by 1, we should proceed more agreeably to truth, and the result would be the same as by the rule.

The truth of the method of proof is obvious; for it is plain that the difference of two numbers added to the less must equal the greater. To prove Subtraction by casting out the 9's, subtract the excess of 9's in the subtrahend from the excess in the minuend, and if the remainder be equal to the excess of 9's in the remainder of the given sum, the work is supposed to be right. N. B. When the excess of the remainder is less than the excess in the subtrahend, 9 must be added to it before subtracting the excesses.

add the remainder 2192, to the subtrahend, 4293, and the sum 6485 being equal to the minuend, the work is right.

	2.	3.	4.	5.
From	3287625	5327467	78213606	12345687
Take	2343756	1008438	27821890	3456289
Rem.	943869			
Proof.	3287625			

Application.

1. Sir I. Newton was born in the year 1642, and died in 1727; how many years did he live? Ans. 85 years.

2. Supposing a man to have been born in 1763, how old was he in 1820? Ans. 57 years.

3. What number is that which taken from 365, leaves 159? Ans. 206.

4. If a man have 125 head of cattle, how many will he have after selling 8 oxen, 11 cows, 9 steers and 13 heifers? Ans. 84.

5. Supposing you lend a neighbor 765 dollars, and he pays you at one time, 86 dollars, and at another, 125 dollars; how much is there yet due? Ans. 554.

6. If you lend a friend 3646 dollars, and he afterwards pay you 2998 dollars; how much is still due? Ans. 648 dolls.

7. What number is, that to which if you add 643, it will become 1826? Ans. 1183.

8. How many years from the flight of Mahomet in 622, to A.D. 1826? Ans. 1204 years.

9. America was discovered by Columbus in 1492, how many years since?

10. A owed B 4850 dollars, of which he paid at one time 200 dollars, at another 475, at another 40, at another 1200, and at another 156; what remains due? Ans. 2779 dollars.

11. The sum of two numbers is 64892, and the greater number is 46234, what is the smaller number? Ans. 18658.

12. America was discovered in 1492, and the settlement of New-England was commenced 128 years afterwards; in what year was it commenced? Ans. 1620.

QUESTIONS.

1. What is Simple Subtraction?
2. What are the given numbers called?
3. What is the difference between them called?
4. How do you write down the numbers for Subtraction?
5. Where do you begin to subtract?
6. What is to be done when the figure in the lower line is larger than the one over it?
7. In such cases what do you do to the next figure in the lower line?
8. How do you prove Subtraction?

3. Simple Multiplication.

SIMPLE MULTIPLICATION is the method of finding the amount of a given number, by repeating it any proposed number of times; as, 6 repeated 4 times, or 4 times 6 is 24.*

In Multiplication there must be at least two numbers given to find a third.

The two given numbers spoken of together are called *factors.* Spoken of separately, the number to be repeated, or multiplied, is called the *multiplicand,* the number by which it is repeated, or multiplied, is called the *multiplier,* and the number found by the operation is called the *product.*

Before the scholar can proceed in this rule, the following table must be thoroughly committed to memory.

Multiplication and Division Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	[illegible]	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Use of the preceding Table for Multiplication.—Find the multiplier in the left hand column and the multiplicand in the upper line; the product will be found in the line with the multiplier, directly under the multiplicand. Thus 48 the product of 6 and 8, is found in the line with 6 under 8.

For Division.—Find the divisor in the left hand column, run your eye along to the right hand till you find the dividend, and right over it in the upper line is the quotient. Thus 48 divided by 6 the quotient is 8.

---

* Multiplication is only an abridged method of Addition, and all the questions in Multiplication may be solved by that rule. Thus 6 multiplied by 4, is the same as 4 sixes added together, or $6\times4=6+6+6+6=24$ But the solution by addition would be extremely tedious, particularly when the multiplier is a large number.

Rule.*

1. Write the multiplier under the multiplicand, with units under units, tens under tens, and so on, and draw a line under them.

2. Begin at the right hand and multiply all the figures of the multiplicand separately by each figure of the multiplier, setting the first figure of the product directly under the figure of the multiplier, which is employed, and carrying for the tens as in Addition.

3. Add these several products together, and their sum is the total product, or answer required.

Proof.†

Make the former multiplicand the multiplier, and the former multiplier the multiplicand, and proceed as before; if the product be equal to the former, the product is right.

Examples.

1. Multiply 376 by 4.

3 7 6
4
1 5 0 4

Here 4 times 6 is 24, which being 2 tens and 4 over, write down the 4 and say 4 times 7 is 28 and 2 carried, is 30; write a cipher and say again 4 times 3 is 12, and 3 carried is 15, which write down, and the work is done.

2. Multiply 43 by 25.

	OPERATION.	PROOF.
	4 3	2 5
	2 5	4 3
	2 1 5	7 5
	8 6	1 0 0
Ans.	1 0 7 5	1 0 7 5

---

* When the multiplier is a single digit, multiplying every figure of the multiplicand by that digit, is evidently multiplying the whole by it; and carrying for the tens is only assigning the several parts of the product to their proper places. This must be obvious from the following analysis of the first examples.

		or
Multiplicand.	3 7 6	3 7 6
Multiplier.	4	4
Product.	1 5 0 4	2 4
		2 8 0
		1 2 0 0
		1 5 0 4

Here 4 times 6 is 24, 7 being in the place of tens, is 70, and 4 times 70 is 280, and 3 being in the place of hundreds, is 300, and 4 times 300 is 1200. Here the multiplicand is divided into parts, and each of the parts multiplied by 3. Their product added together amounts to 1504, the same as by the rule.

Where the multiplier consists of more than 1 digit, it is considered to be divided into as many parts as there are digits, and the whole multiplicand being multiplied by each of these parts, is evidently multiplied by the whole multiplier. The product arising from multiplying by the second figure in the multiplier, or the figure in the place of tens, is ten times as great as it would be if that figure occupied the unit's place, and that arising from the third figure one hundred times as great, and so on, and these values are truly expressed by writing the first figure of each product directly under the figure by which we multiply, as will be evident by inspecting the operation; hence the sum of the several products is the product of the whole multiplicand into the whole multiplier.

† This method of proof depends upon the proposition that two numbers being multiplied together, either of them may be made the multiplier, or the multiplicand, and the product will still be the same, which may be thus proved.—Suppose the two factors to be 6 and 3.

1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1,

Now if we write three lines of 1s with six 1s in a line, it is evident that the whole number of 1s will make as many times 6 as there are lines, that is, 3 times 6, and as many times 3 as there are columns, that is, 6 times 3. Hence it

3. Multiply 37934 by 2.	Product.	-	75868
4. Multiply 357 by 56.	Prod.	-	19992
5. Multiply 46891 by 325.	Prod.	-	15239575
6. Multiply 653246 by 408.	Prod.	-	266524368
7. Multiply 5432176 by 1234.	Prod.	-	6703305184
8. Multiply 848329 by 4009.	Prod.	-	3400950961
9. Multiply 99886 by 98.			
10. Multiply 6842 by 2486.			

CONTRACTIONS.

I. *When there are ciphers on the right hand of one or both the factors.*

RULE.

Neglecting the ciphers, proceed as before, and place on the right hand of the product, as many ciphers as were neglected in both the factors.

EXAMPLES.

1. Multiply 3700 by 200.

```
 37
  2
 ---
740000
```

Here neglecting the ciphers, I multiply 37 by 2, and annex four ciphers, the number neglected in the two factors.

2. Multiply 461200 by 72000.

```
      4612
        72

Prod. 33206400000
```

3. Multiply 5036000 by 70300.

```
      5536
       703

Prod. 3540308000000
```

is plain that 3 times 6 are the same number of units, or give the same product, as 6 times 3, and the same may be shown of any other two factors.

There are several other methods of proof. The following by division will be found very convenient after becoming acquainted with that rule.

I. Divide the product by the multiplier, and if it be right the quotient will be equal to the multiplicand.

Another method much practised, is by casting out the nines.

RULE.

II. Cast the nines out of the multiplicand and multiplier; multiply the two excesses together, cast the nines out of their product and write down the excess; then cast the nines out of the product of the sum, and if the excess be equal to the former, the work is supposed to be right.

EXAMPLE.

```
Multiplicand.   357  - -  6
Multiplier.      56  - -  2
               ----      --
               2142      12  -  3
              1785
              -----
Product.      19992 - - - - - - 3
```

I first cast the nines out of the multiplicand and find the remainder to be 6; then out of the multiplier and find the remainder 2; these being multiplied together, the product is 12, and the excess 3. I then cast the nines out of the product, and find the excess there to be 3 also. Hence I conclude the work is right. This method may generally be depended upon, but it is liable to the same inconvenience as in Addition, that a wrong operation sometimes appears to be right, and for the reason mentioned under that rule. There are other methods of proving Multiplication, but these are deemed sufficient.

II. *When the multiplier is a composite number.*

A *composite number* is one which is produced by the multiplication of other numbers, and the *component parts* are the numbers employed in producing the composite number. Thus 4 times 6 is 24. Here 24 is a composite number, and 4 and 6 are its component parts.

Rule.*

Multiply by one of the component parts, and that product by the other.

EXAMPLES.

1. Multiply 2478 by 36.

```
     2 4 7 8   Because 6 times
           6     6 is 36
    ---------
   1 4 8 6 8
           6
    ---------
Pro. 8 9 2 0 8
```

2. Multiply 8462 by 56. Ans. 473872.

3. Multiply 59375 by 35. Ans. 2078125.

III. *When the multiplier is 9, 99, or any number of nines.*

Rule.†

Annex as many ciphers to the multiplicand as there are nines in the multiplier, and from the sum thus produced, subtract the multiplicand, the remainder will be the product.

EXAMPLES.

1. Multiply 478 by 99.

```
          4 7 8 0 0
              4 7 8
          ---------
Product.  4 7 3 2 2
```

2. Multiply 6473 by 999. Ans. 6466527.

3. Multiply 99 by 9. Ans. 891.

---

* The reason of this rule is obvious; for in the first example, the product of 6 times 2478 multiplied by 6, is as evidently 36 times 2478, as 6 times 6 is 36. A composite number may have 2, 3, or more component parts. Thus 30 is a composite number whose component parts may be 6 and 5, or 3 and 10, or 5, 3 and 2, &c.

† The reason of this rule will appear by considering that annexing a cipher to any sum, is the same as multiplying it by 10, annexing two ciphers the same as multiplying by 100, &c. Now when the multiplier is 9, annexing a cipher to the multiplicand, multiplies it by 10, which repeats it once more than is proposed by the multiplier; therefore if we take the multiplicand from this sum, we have the amount of the multiplicand nine times repeated, or the product arising from multiplying by 9. When there are two nines in the multiplier, annexing two ciphers to the multiplicand multiplies it by 100, which repeats it once more than proposed by the multiplier. Hence, taking the multiplicand once from this sum, we have the true product arising from multiplying it by 99, and the same reasoning is applicable to any number of nines.

Application.

1. If a man earn 3 dollars per week, how much will he earn in a year, which is 52 weeks?
Ans. 156 dollars.

2. If a man thrash 9 bushels of wheat per day, how much will he thrash in 29 days?
Ans. 261 bushels.

3. In a certain orchard there are 26 rows of trees, and 26 trees in a row; how many trees are there in the orchard?
Ans. 676.

4. In dividing a certain sum of money among 352 men, each received 17 dollars; what was the sum divided? Ans. $5984.

5. A certain city is divided into 12 wards, each ward consists of 2000 families, and each family of 5 persons; what is the number of inhabitants in the city?
Ans. 120,000.

6. If a man's income be 1 dollar per day, what will be the amount of his income in 45 years, allowing 365 days to a year?
Ans. 16425 dollars.

7. A certain brigade consists of 32 companies, and each company of 86 soldiers; how many soldiers are there in the brigade?
Ans. 2752.

8. If a person count 180 in a minute, how many will he count in an hour? Ans. 10800.

9. A man had 2 farms, on one he raised 360 bushels of wheat, and on the other 5 times as much; how much did he raise on both?
Ans. 2160.

10. I sold 742 thousand of boards at 24 dolls. per thousand, what did they come to?
Ans. 17808 dollars.

11. Says Tom to Dick, You have only 77 chesnuts, but I have seven times as many; how many have I? Ans. 539.

12. If 4 bushels make a barrel of flour, and the price of wheat be one dollar a bushel, what will 225 barrels of flour cost?
Ans. 900 dollars.

13. Forty-seven men shared equally in a prize, and received 25 dollars each; how much was the prize? Ans. 1175 dolls.

14. Multiply 308879 by twenty thousand five hundred and three. Ans. 6332946137.

15. An army is drawn up in a solid body, and the number of rank and file is equal, being 69 each; what is the whole number of them? Ans. 4761.

QUESTIONS.

1. What is Simple Multiplication?
2. How many numbers must there be given to perform the operation?
3. What are the given numbers called, spoken of together?
4. What are they called, spoken of separately?
5. What is the number found by the operation called?
6. What is the first step in the rule?
7. What the second step?
8. What the third?
9. What is the method of proof?
10. When there are ciphers at the right hand of one or both the factors, how do you proceed?
11. What is a composite number?
12. What are its component parts?
13. How do you proceed when the multiplier is a composite number?
14. How do you proceed when the multiplier is 9, 99, or any number of nines?

4. Simple Division.

SIMPLE DIVISION is the method of finding how many times one number is contained in another of the same denomination. Thus if it were required to know how many times 6 is contained in 18, the answer is 3 times.

As in Multiplication, there are always two numbers given to find a third.

The largest number, or number to be divided, is called the *dividend*. The number by which you divide, is called the *divisor*. The result of the operation, or number of times the divisor is contained in the dividend, is called the *quotient*.

If there be any left after performing the operation, that excess is called the *remainder*. The remainder is always less than the divisor, and is of the same kind as the dividend.

Rule.*

1. Having written the divisor at the left hand of the dividend, find

---

* As Multiplication performs the office of many additions, so does Division perform the office of many subtractions. Hence Division is only an abridgment of Subtraction. Thus the result is the same, whether we divide 14 by 4 three times, or subtract 4 three times from it. Each shows that 14 contains 4 three times, and that 2 remains.

† By the rule the dividend is resolved into parts, and there is found the number of times the divisor is contained in each of these parts, and the sum of these several quotients is the true quotient. This will appear plain from the following example and observations.

```
Divisor. 25)4568 Dividend.
           4500  -   100
 25×100=2500
        ----
        2000
    Add   60
        ----
        2060  - -  80
  25×80 - 2000
        ----
          60
           8
          --
          68
  25×2  -  50
          --      ---
   Rem. 18 Quot. 182
```

Here the dividend is divided into 3 parts, the first is 4500, the second 60, and the third 8. The first part of the dividend employed is 45, but its true value is 4500, and instead of seeking how many times 25 in 45, we in fact seek how many times 25 in 4500, and the quotient, instead of 1, is 100, and the remainder 2000. To this remainder add the second part, and the sum divided by 25, the quotient, instead of 8, is 80, and the remainder 60. Again, to this remainder bring down the other part 8, making it 68, and dividing it by 25, the quotient is 2, and 18 remainder. Now the sum of these several quotients, 182, is the proper quotient arising from dividing 4568 by 25, and 18 remainder, as in example second.

From the preceding example and observations, we perceive the complete value of the first part of the dividend is 10, 100, or 1000, &c. times the value of which it is taken by the rule, according as there are 1, 2 or 3, &c. figures on the right hand, and also that the value of the quotient figure is the same number of times its simple value as the part the dividend employed. Hence to express the true value of any quotient figure, annex as many ciphers as there are places in the dividend, at the right hand of the part employed in obtaining that quotient figure; but as a figure is added to the quotient for each figure brought down in the dividend, at the last division the quotient is complete.

The method of proof must be sufficiently obvious; for as the quotient is the

how many times it is contained in as many of the left hand figures of the dividend as will contain it once or more, and place the answer as the highest figure in the quotient.

2. Multiply the divisor by the quotient figure, and set the product under the part of the dividend used.

3. Subtract the product from the part of the dividend used.

4. Bring down the next figure in the dividend to the right of the remainder, and divide the sum as before. If this sum be less than the divisor, place a cipher in the quotient, and bring down another figure.

Proof.

Multiply the divisor by the quotient, and to the product add the remainder, if any, and if the sum equal the dividend, the work is right.

Examples.

1. Divide 147 by 4.

Divisor. Dividend. Quotient.

```
4 ) 1 4 7 ( 3 6
    1 2
    ---
      2 7
      2 4
      ---
        3 Remainder.*
```

Having written down the dividend and included it within the reversed parenthesis, with the divisor, (4) at the left hand, assume as many figures in the dividend as will contain it once or more. Here it is necessary to assume the two first figures, (14) because 4 is not contained in the first figure, (1). But 4 is contained in 14 three times; therefore, write 3 for the first quotient figure, and multiplying 4, the divisor, by it, place the product, (12) under 14 in the dividend. Subtract 12 from 14 and to the remainder, 2, bring down the next figure 7, making it 27. Again, how many times is 4 contained in 27; 6 times: place 6 in the quotient, multiply the divisor, 4, by it, and the product, 24, place under 27, and subtract, and the work is done. Thus we find that 4 is contained in 147, 36 times, and that 3 remains.

---

number of times the dividend contains the divisor, the product of the quotient and divisor is evidently equal to the part of the dividend exhausted by dividing; and if there be a remainder, or part of the dividend which was not exhausted, it is plain that it must be added to the product of the divisor and quotient to obtain a sum equal to the dividend. There are several other methods of proving Division. The following is a very expeditious way of doing it by casting out the 9's. Cast the 9's out of the divisor and quotient, multiply the excesses and add the remainder to the product, if any. Cast the 9's from the sum, and also from the dividend, and if the two excesses agree, the work is right. This method is liable to the same inconvenience here as in the preceding rules.

* When there is no remainder after division, the dividend is completely exhausted, and the quotient is the complete answer. But when there is a remainder, that would give a part of another unit in the quotient. If the remainder be one fourth, one half, or three fourths as large as the divisor, one fourth, one half or three fourths of the divisor is contained in the dividend in addition to the figures already found in the quotient. Therefore, to express the true value of any remainder, write it after the quotient over a horizontal line, with the divisor under it. The quotient in the first example is expressed thus, 36$\frac{3}{4}$. This $\frac{3}{4}$ is called a Vulgar Fraction, and 36$\frac{3}{4}$ a mixed number.

2. Divide 4568 by 25.

```
Divis. Divid. Quotient.
25 ( 4568 ( 182      Proof.
     25              182 Quot.
     ---              25 Divis.
     206             ---
     200             910
     ----           364
      68            ----
      50            4550
      --              18 Rem.
      18            ---- add
                    4568=Div.
```

3. Divide 85608 by 36.
Quot. 2378.

4. Divide 17354 by 86.
Quot. 201. Rem. 68.

If, when you have brought down a figure to the remainder, it is still less than the divisor, place a cipher in the quotient. bring down another figure and proceed as before.

5. Divide 1044 by 9.
Quot. 116.

6. Divide 34748748 by 24.
Quot. 1447864. Rem. 12.

Contractions.

I. *When the Divisor is a single digit*, the operation may be performed mentally without setting down any figures except the quotient. This is called *Short Division*.

Examples.

1. Divide 867 by 3.

```
Divis. Divid.
 3 ) 867
     ---
     289 Quot.
```

Here seek how many times 3 in 8, and finding it 2 times and 2 over, write 2 under the 8 for the first figure of the quotient, and place the 2 which remained before 6, making it 26. Then seeking how many times 3 in 26, the answer is 8 times and 2 over; therefore write the 8 under 6, and place the 2 which remained before the 7, making it 27. Lastly, seek how many times 3 in 27, and the answer is 9, which write under the 7, and the work is done.

2. Divide 78904 by 4.

```
4 ) 78904
    -----
    19726 Quotient.
        4
    -----
    78904 Proof.
```

3. Divide 234567 by 9.
Quot. 26063.

4. Divide 279060 by 7.

```
       7 ) 279060
           ------
Quot.       39865 Rem. 5.
                7
           ------
Proof.     279060
```

5. Divide 29702 by 6.
Quot. 4950. Rem. 2.

II. *To divide by any number with ciphers at the right hand.*

Rule.

Cut off the ciphers from the divisor, and as many figures from the right hand of the dividend; making use of the remaining figures, divide as usual, and place the figures cut off from the dividend at the right hand of the remainder.

Examples.

1. Divide 36556 by 3200.

```
32|00)365|56(11 Quo.
      32
      --
       45
       32
       --
       1356 Rem.
```

Here I cut off the two ciphers and also 56, and divide 365 by 32, and find the quotient, 11, and remainder, 13; I then bring down the 56 to the right of 13 for the whole remainder, (1356) and the work is done.

2. Divide 7380964 by 23000.
Quot. 320. Rem. 20964.

3. Divide 6095146 by 5600.
Quot. 1088. Rem. 2346.

III. *When the divisor is a composite number.*

Rule.

Divide continually by the component parts instead of the whole divisor at once.

Examples.

1. Divide 31046835 by $56=7\times8$.

```
7)31046835
  --------
8)4435262  1
  -------
Quot. 554407
```

Rem. $6\times7+1=43$.*

2. Divide 7014596 by $72=9\times8$.
Quot. 97424. Rem. 68.

3. Divide 84874 by $48=6\times8$.
Quot. 1768. Rem. 10.

Application.

1. Divide 30114 dollars equally among 63 men; how many dollars must each man receive?
Ans. 478.

2. Supposing a man's income to be 1460 dollars a year; how much is that per day?
Ans. $4.

3. A man dies leaving an estate of 7875 dollars to his 7 sons, what is each son's share?
Ans. 1125.

4. If a field of 34 acres produce 1020 bushels of corn, how much is that per acre?
Ans. 30 bushels.

5. A privateer of 175 men took a prize worth 20650 dollars, of which the owner of the privateer had one half, and the rest was divided equally among the men. What was each man's share?
Ans. $59.

To get one half divide by 2.

* The first remainder, 1, is evidently a unit of the given dividend, but the second remainder, 6, is so many units of the second dividend, and a unit in the second dividend is plainly equal to 7 units in the first. Therefore to find the true remainder, multiply the last remainder by the last divisor but one, and add the preceding remainder, the sum will be the true remainder if there are but two divisors; but if more than two, multiply this sum by the next preceding divisor, and so on till you arrive at the first divisor.

6. What number must I multiply by 25 that the product may be 625? Ans. 25.

7. If a certain number of men by paying 33 dollars each, paid 726 dollars, what was the number of men? Ans. 22.

8. The polls in a certain town pay 750 dollars, and the number of polls is 375; what does each poll pay? Ans. $2.

9. If 45 horses were sold in the West Indies for 9900 dollars, what was the average price of each? Ans. $220 each.

10. On a muster day, 9450 lbs. of beef were provided to dinner 27 regiments, which consisted of 7 companies each, and each company of 100 men; how much was each man's share? Ans. 8 oz.

QUESTIONS.

1. What is Simple Division?
2. How many numbers are necessary to perform the operation?
3. What is the given number called?
4. What is the number sought called?
5. If there be any left after performing the operation, what is it called?
6. Of what kind is the remainder?
7. What is the first part of the rule? the second? the third? the fourth?
8. How do you prove Division?
9. If after bringing down a figure, the sum be still less than the divisor, what is to be done?
10. What is Short Division?
11. How do you divide when there are ciphers on the right hand of the divisor?
12. What do you do with the figures cut off from the dividend?
13. How do you proceed when the divisor is a composite number?

Miscellaneous Examples.

1. The first settlement of New-England was commenced in 1620; how many years from that time to the declaration of Independence in 1776? Ans. 156 years.

2. What number shall I multiply by 8, that the product may be 552? Ans. 69.

3. There are two numbers, the greater is 27 times the less, and the less is a 9th part of 27; what is the greater? Ans. 81.

4. If 9000 men march in a column of 750 deep, how many march abreast? Ans. 12.

5. What is the sum of 16, added to the difference of twice five and twenty, and twice twenty-five? Ans. 36.

6. A was born when B was 26 years old; how old will A be when B is 47? Ans. 21.

7. The remainder of a division is 325, the quotient 467, and the divisor 43 more than the sum of both; what is the dividend? Ans. 390270.

8. If a man's income be 730 dollars a year, what is that per day? Ans. 2 dollars.

9. A gentleman left his son 725 dollars more than his daughter, whose fortune was 15 thousand, 15 hundred and 15 dollars; what was the amount of the whole estate? Ans. 33755 dollars.

SECTION II.

COMPOUND RULES.

1. TABLES

OF THE RELATIVE VALUE OF THE DIFFERENT DENOMINATIONS OF COIN, WEIGHT AND MEASURE, WHICH ARE USED IN THE TRANSACTION OF BUSINESS.

Those which are essentially necessary to be understood by every individual, are expressed in the text, and should be perfectly committed to memory. Those which are less common, and less essential, will be found in the notes.

1. *FEDERAL MONEY.**

10 mills, *m.*	make one	cent, *marked ct.*	
10 cents		dime,	" *d.*
10 dimes, or 100 cts.		dollar,	" $
10 dollars		eagle,	" E.

2. *ENGLISH MONEY.*†

4 farthings, *qr.*	make one	penny, *marked d.*	
12 pence		shilling,	" *s.*
20 shillings		pound,	" £

3. *TROY WEIGHT.*‡

24 grains, *grs.*	make one	pennyweight, *marked,pwt.*	
20 pennyweights		ounce,	" *oz.*
12 ounces		pound,	" *lb.*

By this weight, are weighed gold, silver, jewels, electuaries and liquors.

4. *AVOIRDUPOIS WEIGHT.*§

16 drams, *dr.*	make one	ounce,	*marked oz.*
16 ounces		pound,	" *lb.*
28 pounds		quarter of a hundred,	" *qr.*
4 quarters		hundred weight,	" *cwt.*
20 hundred weight		ton,	" *T.*

By this weight, are weighed all things of a coarse, or drossy nature, such as butter, cheese, meat, groceries and metals, except gold and silver.

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* The nature of this currency will be more fully explained hereafter. See Federal Money.

† The value of the several denominations of English money, is different in different places. An American dollar is equal to 4s. 6d. in England, or 5s in Canada and Nova-Scotia, or 6s. in New-England, Virginia, and Kentucky, or 8s. in New-York, Ohio, and North Carolina, or 7s. 6d. in Pennsylvania, New-Jersey, Delaware, and Maryland, or 4s. 8d. in South Carolina, and Georgia. A guinea is 21s. in England, or 28s. in New-England. A moidore in N. E. is 36s.

‡ The weight used by Apothecaries. in compounding their medicines, is the same as Troy weight, having only some different divisions. The following is the table of Apothecaries' weight.

20 grains, *grs.*	make one	scruple, *marked sc.*	
3 scruples		dram,	" *dr.*
8 drams		ounce,	" *oz.*
12 ounces		pound	" *lb.*

§ The practice of grossing, as it is called, that is, of considering the quarter

5. *OF TIME.**

60 seconds, *s.*	make one	minute,	*marked, m.*
60 minutes	make one	hour	" *h.*
24 hours	make one	day	" *d.*
7 days	make one	week	" *w.*
4 weeks	make one	lunar month	" *mo.*
13 mo. 1 d. 6 h. or 365 d. 6 h.	make one	Julian year	" *Y.*

6. *CIRCULAR MOTION.*

60 seconds ″	make one	minute,	*marked* ′
60 minutes	make one	degree	" °
30 degrees	make one	sign	" s.
12 signs, or 360°	make one	circle.	

Every circle, without regard to its size, is supposed to be divided in 360 equal parts, called degrees; and these again to be subdivided into minutes and seconds; so that the absolute quantity expressed by any of these denominations, must always depend upon the size of the circle.

7. *CLOTH MEASURE.*†

$2\frac{1}{4}$ inches, *in.*	make one	nail,	*marked na.*
4 nails, or 9 in.	make one	quarter	" *qr.*
4 quarters	make one	yard	" *yd.*
5 quarters	make one	English ell	" *E. E.*

to be 28lbs. and the hundred weight to be 112 lb. is now pretty generally laid aside, and, both in buying and selling, 25 lb. are, as they should be, considered a quarter, and 100lbs. a hundred weight. 144 lb Avoirdupois equals 175 lb. Troy; and 192 oz. Avoirdupois equals 175 oz. Troy. An Avoirdupois pound weighs 7000 grains; a Troy pound, 5760 grains. A firkin of butter is 56 lb. A firkin of soap, 64 lb. A barrel of pot ashes, 200 lb. A barrel of candles, 120 lb. A barrel of soap, 265 lb. A barrel of butter, 224 lb. A barrel of pork or beef is 220 lb. A quintal of fish, 112 lb.

* In civil reckoning, the year is divided into 12 Calendar months, and the number of days in each, may be readily called to mind by the following verse.

Thirty days hath September, April, June and November;
February twenty-eight alone, and all the rest have thirty-one.

Another day is added to February every fourth year, making the month consist of 29 days. This is called Bissextile, or Leap-year. Leap-year is found by dividing the year of our Lord by 4; if nothing remain, it is Leap-year; but if 1, 2 or 3 remain, it is 1st, 2d, or 3d after Leap-year. The true solar year consists of 365 days, 5h. 48m. 57s. or nearly 365 1-4 days. A common year consists of only 365 days, and one day is added in Leap-years to make up the loss of one quarter of a day in each of the three preceding years. This method of reckoning was ordered by Julius Cæsar 40 years before the birth of Christ, and is called the Julian Account, or *Old Style.* But as the true year fell 11m. 3s. short of 365 1-4 days, the addition of a day to every fourth year was too much by 44m. 12s. This amounted to 1 day in about 130 years. To correct this error, in 1581, Pope Gregory ordered 10 days to be struck out of the Calendar, by calling the 5th of October the 15th, and to prevent its recurrence, ordered that each succeeding century divisible by 4, should be a Leap-year, and each not divisible by 4, should be a common year. This is called the Gregorian, or *New Style.* The difference between the New Style and Old, is now 12 days.

† A Flemish-ell is 3 qrs. a French-ell, 6 qrs. a Scotch-ell, 37 1-5 inches, and a Spanish var, 33 inches.

8. *LONG MEASURE.**

3 barley corns, *bar.*	make one	inch,	*marked in.*
12 inches		foot	" *ft.*
3 feet		yard	" *yd.*
5½ yds. or 16½ ft.		rod, or pole,	" *rd.* or *po.*
40 rods		furlong	" *fur.*
8 furlongs		mile	" *mile.*
60 geographical, or 69 1-5 English miles		degree	" *deg.*
360 degrees		circumference of earth.	

The use of this measure is to find the distance of places, or to measure any thing where length only is concerned, without regard to breadth.

9. *SQUARE MEASURE.*†

144 inches	make one	square foot.
9 feet		" yard.
30¼ yards, or 272¼ feet		" rod.
40 rods		" rood.
4 roods		" acre.
640 acres		" mile.

This measure is employed in measuring land, and all things where length and breadth are concerned.

10. *SOLID MEASURE.*‡

1728 inches	make one	foot.
27 feet		yard.
40 ft. of round, and 50 ft. hewn timber		ton.
128 ft. i. e. 8 in length, 4 in breadth and 4 in height		cord of wood.

By this, are measured all things which have length, breadth and thickness.

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* For measuring considerable distances, as in surveying, a chain 4 rods in length, and divided into 100 links, or a half chain 2 rods long, and divided into 50 links, is usually employed. In this measure, 25 links make one rod, 100 links one chain, 10 chains one furlong, and 8 furlongs one mile. In addition to the above, in long measure, 60 geographical miles make a degree, 6 points make a line, 12 lines an inch, 4 inches a hand, 3 hands a foot, 5 feet a pace, 6 feet a fathom. A mile is equal to 320 rods, or 1760 yards, or 5280 feet. A league is equal to 3 miles.

† The numbers in this table are produced from the preceding by multiplying the several numbers into themselves, as 12 times 12 is 144, 3 times 3 is 9, &c. We may further observe, that an acre is equal to 160 square rods, or 4840 yards, or 43560 feet.

‡ This is also called cubic measure. For the ease of reckoning, the cord of wood is sometimes called 8 feet. In this case, 4 feet in length, 4 in breadth, and one in height, equal to 16 solid feet, is called a foot, or 8 in length, 4 in breadth, and 6 inches in height, a foot; that is, 1-8 of a cord is called 1 foot, 2-8, 2 feet, &c.

11. *WINE MEASURE.**

4 gills, *gs.*	make one	pint,	*marked*	*pt.*
2 pints		quart	"	*qt.*
4 quarts		gallon	"	*gal.*
42 gallons		tierce	"	*tier.*
63 gallons		hogshead	"	*hhd.*
2 hogsheads		pipe	"	*P.*
2 pipes		tun	"	*T.*

By wine measure are measured rum, brandy, gin, perry, cider, mead, vinegar and oil.

12. *ALE AND BEER MEASURE.**

2 pints, *pts.*	make one	quart,	*marked*	*qt.*
4 quarts		gallon	"	*gal.*
8 gallons		firkin of ale	"	*A. f.*
9 gallons		firkin of beer	"	*B. f.*
2 firkins		kilderkin	"	*kil.*
2 kilderkins		barrel	"	*bar.*
3 kilderkins		hogshead	"	*hhd.*
3 barrels		butt	"	*butt.*

13. *DRY MEASURE.*‡

2 pints, *pts.*	make one	quart,	*marked*	*qt.*
8 quarts		peck	"	*pk.*
4 pecks		bushel	"	*bu.*
8 bushels		quarter	"	*qr.*
4 quarters		chaldron	"	*ch.*

By this measure, all dry goods, as corn, grain, salt, fruit, roots, &c. are measured.

---

* The other denominations of this measure, which are sometimes used, are anchors, runlets and half-hogsheads. An anchor of brandy is 10 gals. a runlet is 18 gals. a half-hogshead, 31 1-2 gals. A pint, wine measure, is equal to 28 7-8 cubic inches, and a gallon to 231 cubic inches. A common cider barrel contains from 32 to 36 gallons.

† A pint in ale and beer measure, is 35 1-4 cubic inches, and a gallon is 282 cubic inches.

‡ The following denominations of this measure are sometimes used: 2 quarts make one pottle, marked *pot.* 2 pottles one gallon, 2 gals. one peck, 4 pks. one bushel, 2 bushels one strike. 2 strikes one coom, 2 cooms one quarter, 5 quarters one load, or wey. A gallon, dry measure, is 268 4-5 cubic inches. A Winchester bushel is 18 1-2 inches diameter, and 8 inches in depth. By act of the Legislature, 1816, the bushel in Vermont, for measuring coal, ashes and lime, was ordered to contain 38 qts or 2553 3-5 cubic inches. A common bushel of 4 pecks, contains 2150 2-5 cubic inches.

The inconveniences which result from so great a diversity in weights and measures, have already engaged the attention of Congress, and we ardently hope that they in their wisdom, may devise some method of removing the evil. and establishing a uniformity. By doing this, they will at the same time promote the welfare of the people, and render an essential aid to the cause of science.

Note.—20 particular things make one score, 12 one dozen, 12 dozen one gross, 12 gross one great gross. The habit of reckoning by dozens originated out of the English method of reckoning money; articles which were 4s. a dozen, being 4d. a piece, 3s. a doz. 3d. a piece, &c.

2. Compound Addition.

Compound Addition is the adding of numbers which consist of different denominations, as pounds, shillings, pence and farthings. The operations in this and the four following rules, are to be regulated by the values expressed in the 13 preceding tables.

Rule.*

Place the numbers so that those of the same denomination may stand directly under each other.

Add the lowest denomination and carry for that number which it takes for that denomination to make one of the next higher, setting down the remainder. Proceed with the columns of each denomination in the same way till you come to the last, which is to be added as in Simple Addition.

Proof.

The method of proof is the same as in Simple Addition.

Examples.

1. FEDERAL MONEY.

1. What is the sum of 4E. $5. 3d. 4cts. 6m. and 12E. $0. 9d. 8cts. 7m. added together?

OPERATION.

E.	$	d.	cts.	m.
4	5	3	4	6
12	0	9	8	7
16	6	3	3	3

As the value of the denominations increase in a tenfold proportion, we here carry by 10 from one denomination to another, and the operation is performed precisely as in Simple Addition. In business, we commonly reckon by dollars and cents only, as in the third and fourth examples.

2.

E.	$	d.	cts.
25	8	2	6
12	8	7	4
13	2	1	3
51	9	1	3
26	0	8	7
51	9	1	3

3.

$	cts.
1259	43
2568	74
7289	93
11118	10
9858	67
11118	10

4.

$	cts.
2228	53
3262	44
120	36

2. ENGLISH MONEY.

1. What is the sum of £9 16s. 10d. £7 10s. 9d. and 18s 6d. when added together? Ans. £18 6s. 1d.

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*The reason of this rule must be evident from what has been already said; for in carrying from one denomination to another, we only express in the higher, the value which we omitted to write down in the lower.

OPERATION.

	£	s.	d.
	9	16	10
	7	10	9
		18	6
Sum.	18	6	1
	8	9	3
Proof.	18	6	1

Having written down the numbers as the rule directs, begin with the lowest denomination, (pence) and say, 6 to 9 is 15, and 10 is 25. Now since 12d. make a shilling, you must carry by 12; and as there is twice 12 in 25, and 1 over, therefore, set down the 1, and carry 2 to the shillings, and say 2 carried to 8 is 10, and 6 is 16, and 10 is 26, and 10 is 36, and 10 is 46, (the 1's in the second column of shillings being so many tens,) and 20, the number of shillings in a pound, being contained in 46 twice and 6 over, write down the 6, and carry 2 to the pounds, and say, 2 to 7 is 9, and 9 is 18, which write down, and the work is done.

The method of proof must be sufficiently obvious from the example and what has been already taught in Simple Addition.

2.

£	s.	d.	qrs.
47	7	6	2
3	9	4	3
5	13	9	1
4	11	11	0

3.

£	s.	d.	qrs.
21	18	8	0
48	10	10	3
13	16	4	2
9	0	6	1

4.

£	s.	d.
128	18	11
327	19	9
813	14	10
212	15	8

3. TROY WEIGHT.

	lb.	oz.	pwt.	grs.
	17	3	15	11
	13	2	9	16
	15	6	10	8
	12	10	4	6
Sum	58	10	19	17
	41	7	4	6
Prf.	58	10	19	17

lb.	oz.	pwt.	grs.
14	10	13	20
13	10	18	21
14	10	10	10
10	1	2	3

lb.	oz.	pwt.	grs.
27	11	17	17
17	10	13	13
13	11	13	11
10	0	1	2

4. AVOIRDUPOIS WEIGHT.

	cwt.	qr.	lb.	oz.	dr.
	15	2	15	15	15
	10	1	8	9	11
	12	2	10	12	1
	8	3	20	6	5
Sum.	47	1	27	12	0
	31	3	11	12	1
Prf.	47	1	27	12	0

T.	cwt.	qr.	lb.	oz.
2	16	1	15	8
2	12	2	10	8
1	10	3	6	4
2	8	1	11	4

T.	cwt.	qr.	lb.	oz.
3	12	2	10	2
1	13	0	11	1
2	14	1	10	8
1	15	0	4	12

5. TIME.

	w.	d.	h.	m.	s.
	3	6	18	40	42
	2	4	12	45	30
	1	6	18	15	15
	0	4	12	40	15
Sum.	9	1	14	21	42
	5	1	19	41	00
Proof.	9	1	14	21	42

Y.	mo.	w.	d.
18	10	2	5
12	6	1	6
5	8	0	4
7	10	3	2

Y.	mo.	w.	d.	h.	m.	s.
17	11	3	6	23	30	45
12	9	2	4	12	46	12
8	6	1	0	20	15	0
10	4	0	4	0	45	30

6. CIRCULAR MOTION.

	°	′	″
	25	17	18
	17	49	56
	6	35	24
	10	17	16
Sum.	59	59	54
	34	42	36
Proof.	59	59	54

s.	°	′	″
2	10	45	30
4	15	40	09
3	4	26	10
1	0	11	45

s.	°	′	″
5	0	0	0
0	18	50	55
2	16	59	41
	4	2	0

7. CLOTH MEASURE.

yds.	qrs.	n.
320	3	2
18	1	0
112	2	1
10	0	3
614	3	2
141	0	0
461	3	2

yds.	qrs.	n.
614	3	3
36	1	2
7	0	1
1	2	0

E.E.	qrs.	n.
18	4	2
26	2	3
10	1	2
12	3	1

8. LONG MEASURE.

deg.	mi.	fur.	rd.	ft.	in.	bar.
168	57	7	26	15	11	2
124	53	6	18	7	6	1
79	36	1	7	9	10	0
4	7	3	0	3	2	1
377	35	2	13	3	6	1
208	37	2	26	3½	6	2
377	35	2	13	3	6	1

mi.	fur.	rd.	yds.	ft.	in.	bar.
37	3	14	2	2	5	1
28	4	17	1	1	10	2
17	4	4	3	1	6	0
10	7	20	4	0	2	1

9. SQUARE MEASURE.

rds.	ft.	in.
36	179	137
19	248	119
12	96	75
18	110	122
87	91½	21
50	183¾	28
87	91½	21

yds.	ft.	in.
28	7	119
9	3	75
29	6	120
4	8	12

acr.	roods.	rds.	ft.	in.
756	3	37	245	128
29	1	28	13	110
116	2	18	128	16
42	0	4	116	112

10. SOLID MEASURE.

yds.	ft.	in.
75	22	1412
18	26	195
4	8	1091
16	12	1110
115	16	352

cor.	ft.	in.
18	120	1015
24	80	159
40	116	1111
12	64	826

cor.	ft.	in.
678	122	1600
776	114	1560
489	76	860
376	118	1487

11. WINE MEASURE.

hhds.	gals.	qts.	pts.
39	52	3	1
16	27	1	0
35	12	0	1
29	38	2	0
121	4	3	0

hhds.	gals.	qts.	pts.
37	39	3	5
9	52	2	1
4	28	0	0
32	19	1	6

T.	P.	hhds.	gals.	qts.
4	9	4	37	2
6	0	1	30	5
3	7	4	1	3
3	9	0	40	2

12. ALE AND BEER MEASURE.

A.b.	fir.	gal.
49	3	7
20	2	3
9	0	4
17	3	0
97	1	6

B.b.	fir.	gal.
19	3	7
18	1	5
14	2	3
12	1	6

hhd.	gal.	qts.	pts.
120	40	3	1
100	12	2	0
96	16	1	1
427	28	2	0

13. DRY MEASURE.

qr.	bush.	pks	qts.
8	7	3	7
4	6	2	6
16	4	1	2
3	3	3	5
33	6	3	4

bush.	pks.	qts.	pts.
36	3	0	1
18	0	4	0
10	1	3	0
12	2	6	1

ch.	qr.	bu.	pks.
36	2	6	0
24	3	3	2
20	0	7	0
15	1	0	3

3. Compound Subtraction.

COMPOUND SUBTRACTION is the method of finding the difference between numbers which consist of different denominations.

Rule.*

Write the less number under the greater, so that the parts which are of the same denomination, may stand directly under each other. Begin with the last denomination, and if the lower number exceed the one over it, borrow as many units as will make one of the next higher, and subtract it therefrom; to the difference add the upper number, remembering, when you borrow always to carry one to the next superior denomination in the subtrahend.

Proof.

The method of proof is the same as in Simple Subtraction.

Examples.

1. FEDERAL MONEY.

1. Suppose I lent 3E. $8 7d. 4cts. and received 1E. $9, how much remains due?

OPERATION.

	E.	$	d.	cts.
Lent	3	8	7	4
Recd.	1	9	0	0
Due.	1	9	7	4
Proof.	[illegible]	8	7	4

2.

$	cts.
2740	23
1287	94
1452	29

3.

$	cts.
2564	87
1769	44

2. ENGLISH MONEY.

1. Suppose I borrowed £149 10s. 8d. and paid £86 12s. 4d. how much remains to be paid?

	£	s.	d.
Bor.	149	10	8
Paid.	86	12	4
Due.	62	18	4
Proof.	149	10	8

2.

£	s.	d.	qr.
791	9	8	1
197	16	4	2

3.

£	s.	d.
439	9	10
243	12	4

3. TROY WEIGHT.

	lb.	oz.	pwt.	gr.
Bought	440	5	15	20
Sold	60	8	19	12
Rem.	379	8	16	8

lb.	oz.	pwt.	gr.
274	8	12	10
148	4	16	19

* Borrowing in this rule depends upon the same principle as in Simple Subtraction, and differs from it only on account of the numbers being of different denominations. Hence the reason of the rule must be obvious from what has already been said.

4. AVOIRDUPOIS WEIGHT.

1.

	cwt.	qr.	lb.	oz.	dr.
Bought	18	2	20	10	8
Sold	4	3	16	12	4
Rem.	13	3	3	14	4

2.

T.	cwt.	qr.	lb.	oz.	dr.
9	11	3	19	4	11
3	12	1	20	9	7

5. TIME.

1.

mo.	d.	h.	m.	s.
6	17	13	27	19
3	12	16	41	35
3	4	20	45	44

2.

y.	mo.	w.	d.	h.	m.	s.
48	9	2	5	19	27	31
19	9	3	4	20	19	49

6. CIRCULAR MOTION.

1.

°	′	″
120	45	33
80	51	48
39	53	45

2.

s.	°	′	″
8	24	40	12
4	28	50	55

3.

s.	°	′
4	14	16
0	18	44

7. CLOTH MEASURE.

1.

yds.	qrs.	n.
35	1	3
19	1	2
15	3	3

2.

E.E.	qrs.	n.
432	3	1
177	3	2

3.

yds.	qrs.	n.
78	2	3
49	3	2

8. LONG MEASURE.

1.

yds.	ft.	in.
24	2	10
16	1	11
8	0	11

2.

deg.	m.	fur.	rds.	ft.	in.	bar.
36	41	3	22	8	7	2
18	35	5	36	3	9	2

9. SQUARE MEASURE.

1.

A.	roods.	rds.	ft.
29	3	10	156
24	3	25	158
4	3	24	270¼

2.

A.	roods.	rds.	ft.	in.
46	3	19	28	110
18	0	21	105	101

10. SOLID MEASURE.

1.

cord.	ft.	in.
264	105	1106
146	115	1640
117	117	1194

2.

yd.	ft.	in.
79	22	927
27	25	1525

3.

cord.	ft.	in.
216	12	1716
46	106	302

11. WINE MEASURE.

1.

hhd.	gal.	qts.	pts.
68	18	3	1
36	46	2	1
31	35	1	0

2.

T.	P.	hhd.	gal.	qts.	pts.	gs.
8	2	0	48	1	0	1
4	0	3	24	3	1	0

12. DRY MEASURE.

1.

qr.	bu.	pks.	qts	pts
6	5	2	7	0
4	6	3	7	1
1	6	2	7	1

2.

ch.	qr.	bu.	pk.	qt.	pt.
38	3	4	1	0	1
24	2	6	1	7	0

Application of the two preceding Rules.

1. If a man purchase a yoke of oxen for £15 5. 8d. four cows for £20 10s. 6d. and a horse for £26, what did they all cost?

Ans. £61 16s. 2d.

2. I buy 4 yards of silk for £1 2s. one piece of shirting for £1 0s. 8d. 3 handkerchiefs for 9s. 3d. one yd. of cambrick for 4s. 3½d. and one penknife for 3s. 9d. what is the amount of the purchase?

Ans. £2 19s. 11½d.

3. A man sold a lot of land for £735 11s. 6d. and received at one time £195 13s. 11d. and at another £61 5s.; how much is there yet due?

Ans. £478 12s. 7d.

4. A man bought 624 yds. 3 qrs. of cloth, and sold at one time 247 yds. at another 25 yds. 2 qrs. and at another 114 yds 1qr. how much has he left?

Ans. 238.

QUESTIONS.

1. What is the table of Federal money, and how are the several denominations marked?
2. The table of English money?
3. The table of Troy Weight?
4. Of Avoirdupois Weight?
5. Of Time?
6. Of circular motion?
7. Of cloth measure?
8. Of long measure?
9. Of square measure?
10. Of solid measure?
11. Of wine measure?
12. Of ale and beer measure?
13. Of dry measure?
14. What is Compound Addition?
15. By what are the operations in the Compound Rules regulated?
16. What is the rule for Compound Addition?
17. What is the method of proof?
18. What is Compound Subtraction?
19. What is the rule for Compound Subtraction?
20. What is the method of proof?

4. Reduction.

Reduction is the method of bringing numbers of one name or denomination into another, retaining the same value.

Reduction is of two kinds, *Descending* and *Ascending*.

Reduction Descending is when a higher denomination is to be brought into a lower, as pounds into shillings, pence and farthings, and is performed by Multiplication.

Reduction Ascending is when a lower denomination is to be brought into a higher, as farthings into pence, shillings and pounds, and is performed by Division.

1. *REDUCTION DESCENDING.*

Rule.

Multiply the highest denomination by the number which it takes of the next less to make one of that, adding the number of the second name; and so continue till you have brought it as low as the question requires.

Examples.

1. In £65 4s. 6d. 2qrs. how many farthings?

£.	s.	d.	qrs.
65	4	6	2
20			
1304			
12			
15654			
4			
62618			

Here 65 pounds is the highest denomination. Then because 20 shillings make one pound, I multiply 65 by 20, adding at the same time, the 4s. to the product, and the sum is 1304 shillings; then because 12 pence make one shilling, I multiply the shillings by 12, adding the 6d. and the sum is 15654 pence; lastly, because 4 farthings make one penny, I multiply the pence by 4, adding the 2 farthings, and the work is done. In this way I find that in £65 4s. 6d. 2qrs. there are 62618 farthings.

2. In £1465 14s. 5d. how many farthings? Ans. 1407092.qrs.

3. In 22lb. 6 oz. 10 pwt. 15 grs. how many grains?
Ans. 129855.

4. In 4E. $8 4d. 6cts. 2m. how many mills?
Ans. 48462 mills.*

5. In 250 Eagles how many cents? Ans. 250000 cents.

6. In 37 pistoles, at 22s. how many pence and farthings?
Ans. 9768d. 39072 qrs.

6. In 29 guineas, at 28s. how many farthings?
Ans. 38976 qrs.

---

* To reduce *eagles* to *dollars* add one cipher; to *dimes* add 2; to *cents* add 3; to *mills* add 4. To reduce *dollars* to *dimes* add one cipher; to *cents* add 2; to *mills* add 3. To reduce *dimes* to *cents* add one cipher; to *mills* add 2. To reduce *cents* to *mills* add one cipher.

2. REDUCTION ASCENDING.

Rule.

Divide the lowest denomination by the number which it takes of that to make one of the next higher, and so continue to do till you have brought it into the denomination required.

Examples.

1. In 62618 farthings how many pounds?

```
4 ) 6 2 6 1 8
   -----------
1 2 ) 1 5 6 5 4  2qrs.
      ---------
2|0 ) 1 3 0|4  6d.
      -------
     £6 5   4s.
Ans. £65 4s. 6d. 2qrs.
```

By comparing the examples in Reduction Descending with those in Reduction Ascending, it will be seen that they reciprocally prove each other.

2. In 1407092 farthings how many pounds?
Ans. £1465 14s. 5d.

3. In 20465 mills how many eagles, dollars, &c.
Ans. 2E. $0 4d. 6cts. 5 mills.

4. In 8642 mills how many dollars?
Ans. $8 6d. 4cts. 2m.

5. In 39072 farthings how many pistoles? Ans. 37.

6. In 129855 grains how many pounds?
Ans. 22lb. 6oz. 10pwt. 15grs.

7. In 25684 cents how many eagles?
Ans. 25E. $6 8d. 4cts.

3. REDUCTION ASCENDING AND DESCENDING.

It is thought to be unnecessary to give examples of all the weights and measures. The following will be sufficient to enable the attentive scholar to make a general application of the rules. His judgment will direct him to the tables by which the several examples are reduced.

1. In £151 10s. how many dollars? Ans. $505.

2. In 75 pistoles, at 22 shillings, how many pounds?
Ans. £82 10s.

3. In 78lb. 5oz. 18pwt. how many pennyweights?
Ans. 18838pwt.

4. In 7cwt. 3qr. 10lb. how many drams? Ans. 224768dr.

5. In 2349 pints how many bushels?
Ans. 36bu. 2pk. 6qt. 1pt.

6. In 4 pieces of cloth, each 14 yards, how many nails?
Ans. 896.

When it is required to take quantities of several denominations, each an equal number of times from a given quantity. Rule.—Reduce each of the quantities to the lowest denomination mentioned, and add them together for a divisor; reduce the given quantity to the same denomination for a dividend; and the quotient will be the number sought.

7. In £33 how many guineas, pounds, dollars and shillings, of each an equal number?

```
1 guinea=28s.
1 pound =20s.
1 dollar= 6s.  33
1 shilling=1s. 20
         ------
     55 ) 660 ( 12 Ans.
          55
          --
          110
          110
          ---
```

8. Supposing a man to be 28 years old, how many seconds has he lived, allowing 365 days, 6h. to a year? Ans. 883612800.

9. Reduce one mile to barley corns.

```
     1 mile.
     8
     -
     8 furlongs.
    40
    --
   320 rods.
    5½ *
  ----
  1600
   160
  ----
  1760 yards.
     3
  ----
  5280 feet.
    12
 -----
 63360 inches.
     3
 -----
Ans. 190080 barley corns.
```

10. A man is desirous of drawing off a hogshead of wine into bottles containing gallons, two quarts, quarts and pints of each an equal number, how many must he have? Ans. 33 of each and 9 pints over.

11. How many spoons, each weighing 3oz. can be made from 5lb. of silver? Ans. 20.

12. In 1 ton how many drams? Ans. 573440dr.

13 In 425060 seconds how many hours? Ans. 118h. 4m. 20s.

14. In 63360 feet how many miles?

```
        3 ) 63360
           ------
  5½ † 11 ) 21120 yards.
           ------
            1920
               2
           -----
    4|0 ) 384|0 rods.
          -----
        8 ) 96 furlongs.
           ----
     Ans. 12 miles.
```

15. In £231 16s. how many ducats, at 4s. 9d. each? Ans. 976.

16. How many seconds from the birth of Christ to the year 1824, allowing 365d. 5h. 48m. 57s. to a year? Ans. 57559853088.

17. How many inches from Montpelier to Burlington, it being 38 miles? Ans. 2407680.

18. How many seconds are there in 8 signs, 12° 14′ 26″? Ans. 908066.

19. In 240,000rds. how many acres? Ans. 1500 acres.

---

* To multiply by ½ take half the multiplicand. The several steps in this example present at one view the number of furlongs, rods, yards, feet, inches and barley corns in a mile.

† For 5½ divide by 11 and multiply the product by 2 because 5½=11 halves, and 11 being twice as great as 5½, the true divisor, the quotient will be only half as great as it ought to be; it must therefore be doubled.

20. In 7248 nails how many ells English?
Ans. 362E.E. 2qrs.

21. In 190080 inches how many leagues? Ans. 1.

22. In 1008 quarts of cider how many tuns? Ans. 1.

23. In 25 pistoles at 22s. how many pounds?
Ans. £27 10s.

Miscellaneous Examples.

1. If a man drink a pint of rum a day, how much will he drink in a year? Ans. 45gal. 2qt. 1pt.

2. How many barley corns will reach round the world, supposing it to be 25020 miles? Ans. 4755801600.

3. Divide $120 among 4 men so that the shares shall be to one another as 1, 2, 3, 4. Ans. 12, 24, 36, 48.

4. How many steps of 2 feet 6 inches, must a man take in going from Burlington to Boston, it being 190 miles? Ans. 401280 steps.

5. How many lots, each containing three quarters of an acre, are there in one square mile? Ans. 853 lots, and 40 rods over.

6. How many cubic inches in a cord of wood? Ans. 221184.

7. In £500 how many eagles, moidores, pistoles, crowns and dollars of each an equal number?
Ans. 76 of each, and 908 pence over.

8. If a vinter be desirous to draw off a pipe of wine into bottles containing pints, quarts and 2 quarts, of each an equal number, how many must he have? Ans. 144 of each.

9. There are three fields, one containing 7 acres, another 10 acres and the other 12 acres and 1 rood; how many shares of 76 rods each are contained in the whole?
Ans. 61 shares, and 44 rods over.

10. In 172 moidores at 36s. each, how many eagles, dollars and nine-pences, of each an equal number?
Ans. 92 of each, and 68 nine-pences over.

11. In 470 boxes of sugar, each 26lb. how many cwt.?
Ans. 109cwt. 0qrs. 12lb.

12. If cigars cost one and a half cent each, and a person smoke 3 cigars per day, how much will it cost him for cigars during the months of January, February and March in a common year?
Ans. 405 cents, or $4 5cts.

13. In 5529600 cubic inches, how many cords of wood?
Ans. 25 cords.

QUESTIONS.

1. What is Reduction?
2. Of how many kinds is Reduction?
3. What is Reduction Descending?
4. What is Reduction Ascending?
5. What is the rule for Reduction Descending?
6. What is the rule for Reduction Ascending?
7. How is Reduction proved?
8. How do you proceed when you wish to take quantities of several denominations, each an equal number of times from a given quantity?
9. How many shillings in a guinea? in a moidore? in a crown? in a ducat? in a pistole?
10. Which of the fundamental rules are employed in Reduction?

5. Compound Multiplication.

Compound Multiplication is the method of finding the amount of a given number, consisting of different denominations, by repeating it a proposed number of times.

Rule.*

Write the multiplier under the lowest denomination of the multiplicand. Multiply the several denominations successively by the multiplier, setting down the excess and carrying from each denomination to the next higher, as in Compound Addition.

Proof.

The method of proof is the same as in Simple Multiplication.

Examples.

1. What will 5lb. of tea cost at 1 dol. 2 dimes, 7 cts. per pound ?

$ d. cts.	or $ cts.	or cts.
1 2 7	1 27	127
5	5	5
6 3 5	6 35	635 cts.

By this example it will be seen that the operations by this rule in Federal Money are precisely the same as in Simple Multiplication, one dollar, 2 dimes, 7 cts. being just equal to 1 dol. 27 cts. or to 127 cts. Hence the 127 cents multiplied by 5 the answer is 635 cts.=6 dols. 35 cts.=6 dols. 3 dimes, 5 cts. and the given numbers may in all cases be expressed as a simple number in the lowest denomination mentioned, or as a compound number.

2. What is the cost of 6 lb. of tobacco, at 2s. 6d. 2qrs. per lb ?

s.	d.	qrs.
2	6	2
		6
15s.	3d.	0

Here 6 times 2 is 12, but 12qrs. are equal to 3d. therefore set down 0 and carry 3. Then 6 times 6 is 36 and 3 to carry is 39d.=3s. 3d. set down 3d. and say 6 times 2 is 12 and 3 to carry is 15s. which set down.

3. What will 3lb. of green tea cost, at 9s. 6d. per pound ?
Ans. £1 8s. 6d.

4. What will 6lb. of nails cost at 9 cents per pound ?
Ans. 5 dimes, 4cts. or 54cts.

5. What will 9cwt. of cheese cost, at £1 11s. 5d. per cwt. ?
Ans. £14 2s. 9d.

6. What will 6 cows cost, at £4 6s. 8d. each ?
Ans. £26.

7. What will 5lb. of loaf sugar cost, at 1s. 3d. per pound ?
Ans. 6s. 3d.

8. What will 8 bushels of corn cost, at 5d. 7cts. or 57cts. per bushel ?
Ans. $4 5d. 6 cts. or $4 56cts.

9. What will 9 yards of cloth cost at 5s. 4d. per yard ?
Ans. £2 8s.

10. What will 12 gallons of brandy cost, at 9s. 6d. per gal. ?
Ans. £5 14s.

* The product of a number consisting of different denominations by a simple number, is evidently expressed by the several products of the different parts multiplied by the simple number. Thus, £2 6s. 4d. multiplied by 6, the several products will be £12 36s. 24d.=(by taking the shillings from the pence and the pounds from the shillings and placing them in the shillings and pounds respectively) to £13 18s. 0d. which is agreeable to rule; and the same will be true when the multiplicand is any compound number whatever.

When the multiplier exceeds 12, *and is a composite number*, the component parts may be employed successively, as in Simple Multiplication, instead of multiplying by the whole number at once.

Examples.

1. What will 16 cwt. of cheese cost, at £1 18s 8d per cwt.?

	£	s.	d.
	1	18	8
			4
	7	14	8
			4
Ans.	30	18	8

Here because 16 is produced by multiplying 4 by 4, multiply the price by 4, and that product again by 4.

2. What will 28 yds. of broad cloth cost, at 19s 4d per yard? Ans. £27 1s. 4d.

3. What will 96 quarters of rye cost, at £1 3s 4d per quarter? Ans. £112.

4. What will 63 bushels of rye cost, at 63 cents per bushel? Ans. 3969cts. or $39 69cts.

2. *When the multiplier cannot be produced by the multiplication of two small numbers*, take two such numbers as come the nearest to it, and then find the value of the odd parts and add or subtract as the case requires.

Examples.

1. What will 47 yards of cloth cost, at 17s 9d per yard?

	£	s.	d.
	0	17	9
			5
	4	8	9
			9
	39	18	9
	1	15	6
Ans.	41	14	3

Multiplying by 5 and by 9 gives the price of 45 yds. but this is 2 yds. short of the given quantity. Therefore multiply 17s 9d by 2, and it gives £1 15s 6d for the price of 2 yds. which added to the price of 45 yds. gives the price of the whole.

2. What will 94 pair of silk stockings cost at 12s 2d per pair? Ans. £57 3s. 8d.

3. What will 31 bushels of oats cost, at 25 cents per bush.? Ans. 775cts. or $7 75 cts.

4. What is the weight of 23 silver spoons, each weighing 1oz 9pwts 14grs? Ans. 2lb. 10oz. 0pwt. 10grs.

3. *When the multiplier exceeds* 100, find the cost of 100, multiply it by the number of hundreds, and to this product add the cost of the odd parts and their sum will be the answer required.

Examples:

1. What will 512 bushels of wheat cost, at 5s 10d per bushel?

	5	10	
		10	
2	18	4	price of 10 bushels.
		10	
29	3	4	price of 100 bushels.
		5	
145	16	8	price of 500 bushels.
3	10	0	price of 12 bushels.
£149	6	9	price of 512 bushels.

2. What will 235 bushels of wheat cost, at $1 25 cents per bushel? Ans. 29375cts or $293 75cts. or 29E. $3 7d. 5cts.

3. What will 700 bushels of potatoes cost, at 1s 3d per bush.? Ans. £43 15s.

4. What will 297 yards cost, at 17s 3d 2qrs. per yard? Ans. £256 15s. 7½d.

Duodecimals.

DUODECIMALS are so called because the denominations decrease by 12 from the place of feet towards the right hand, as in the following

TABLE.

A foot	is	12 inches or primes,	*marked*	′
An inch	is	12 seconds	"	″
A second	is	12 thirds	"	‴
A third	is	12 fourths, &c.	"	⁗

Rule.*

Write the several terms of the multiplier under the corresponding terms of the multiplicand; then multiply the whole multiplicand by the several terms of the multiplier successively, beginning at the right hand, and placing the first term of each of the partial products under its respective multiplier, remembering to carry one for every 12 from a lower to the next higher denomination, and the sum of these partial products will be the answer, the left hand term being feet, and those towards the right primes, seconds, &c.

This is a very useful rule in measuring wood, boards, &c. and for artificers in finding the contents of their work.

Examples.

1. How many square feet in a floor 10 feet 4′ long, and 7 feet 8′ wide?

	10f.	4′		
	7	8		
	6	10	8	
	72	4		
Ans.	79f.	0′	8″	

2. How much wood in a load 7ft. 6′ long, 4ft. 8′ wide and 4ft. high?

Ans. 140ft. or 1 cord 12ft.

Multiply the length by the width, and this product by the height.

3. How many square feet in a board 16ft 4in long, and 2ft 8in wide? Ans. 43ft. 6in. 8n.

*The rule may be expressed in general terms thus. When feet are concerned, the product is of the same denomination as the term multiplying the feet; and when feet are not concerned, the name of the product will be expressed by the sum of the indices of the two factors, or of the strokes over them. Thus $4' \times 2'' = 8'''$. Here one of the factors is inches, the other seconds, and the indices or strokes over them amount to 3, hence the product, 8, is thirds. And in the same manner $8'' \times 3'' = 24''''$ or, divide by $12, = 2'''$ The reason of the rule may be shown by the first example. The 4′ are 4 twelfths of a foot and the 8′ are 8 twelfths of a foot, and $\frac{4}{12} \times \frac{8}{12} = \frac{32}{144}$ or $\frac{32}{12}$ of $\frac{1}{12}$ or 32″, which reduced gives 2′ 8″; putting down the 8″ we reserve the 2′ to be added of 10ft. by 8′, or $\frac{8}{12}$, which product is $\frac{80}{12}$, to which 2 being added, we have $\frac{82}{12}$ or 6ft. 10′. Next multiplying 4′ or $\frac{4}{12}$ by 7 we have $\frac{28}{12}$ or 2ft. 4′, which added to the product of 10 by 7 gives 72ft. 4′, and these results added together give 79ft 0′ 8″ for the product. The same reasoning may be extended to cases where there is a greater number of denominations.

4. How many feet in a stock of 12 boards 14ft 6′ long and 1ft 3′ wide? Ans. 217ft. 6in.

Find the content of one board and multiply that by the number of boards, as in Compound Multiplication carrying for 12.

5. What is the content of a ceiling 43ft. 3′ long and 25ft. 6′ broad? Ans. 1102ft. 10′ 6″.

6. How much wood in a load 6ft. 7′ long, 3ft. 5′ high, and 3ft. 8′ wide?

Ans. 82ft. 5′ 8″ 4‴.

7. What is the solid content of a wall 53ft. 6′ long, 12ft. 3′ high and 2ft. thick?

Ans. 1310ft. 9′.

8. How many cords in a pile of 4 foot wood 24ft. long and 6ft. 4′ high? Ans. 4¾ cords.

9. How many square yards in the wainscoting of a room 18ft. long, 16ft. 6′ wide and 9ft. 10′ high? Ans. 324y. 4ft. 6′.

10. How much wood in a cubick pile of wood measuring 8ft on every side? Ans. 4.

11. How many square feet in a platform which is 37 feet, 11 inches long, and 23 feet 9 inches broad? Ans. 900ft. 6′, 3″.

12. How much wood in a load, 8 feet, 4 inches long, 3 feet 9 inches wide, and 4 feet, 5 inches high? Ans. 138ft. 0′, 3″.

13. How many feet of ceiling in a room which is 28 feet, 6 inches long, and 23 feet, 5 inches broad? Ans. 667ft. 4′, 6″.

14. How many square feet are in a board which is 15 feet, 10 inches long and 9¾ inches wide? Ans. 12ft. 10′, 4″, 6‴.

QUESTIONS.

1. What is Compound Multiplication?
2. How are the numbers to be placed?
3. How is the multiplication performed?
4. How, when the multiplier is a composite number?
5. What is a composite number?
6. What is to be done when the multiplier cannot be produced by two small numbers?
7. When the multiplier exceeds 100, how do you proceed?
8. What is the use of Compound Multiplication?
9. How do you prove Compound Multiplication?
10. Why are Duodecimals so called? What is the Table?
11. How do you place the number for multiplication of Duodecimals?
12. Where do you begin to multiply?
13. How are the several products to be set down?
14. What is the use of Duodecimals?

6. Compound Division.

Compound Division is the method of finding how often one number is contained in another of different denominations.

Rule.*

Place the numbers as in Simple Division, and divide the several denominations of the dividend successively by the divisor.

If there be a remainder after dividing any denomination, it must be reduced to the next lower, adding the number in the lower denomination. Divide the sum as usual; and so on till the whole is finished.

Proof.

The method of proof is the same as in Simple Division.

Examples.

1. If 31 bushels of oats cost 7 dollars, 75 cents, what are they per bushel?

$ cts. cts.
7 75=775
cts. cts.
31) 775 (25 Ans.
62
—
155
155
—

Here the cost is reduced to cts. and then the operation becomes the same as in Simple Division.

2. If 9 yards of cloth cost £4 3s 7d 3qrs what is it per yard?
Ans. 9s. 3d. 2qrs.

This and other questions where the divisor is less than 10, may be as conveniently solved by short division. When the number in the highest denomination is less than the divisor, it must be reduced to the next lower before dividing.

3. If 126 lb. of nails cost $10 and 8 cents, what are they per pound? Ans. 8 cents.

4. If 35 yards of cloth cost £57 5s. 5d. what is it per yard?

£ s. d. £ s. d. qrs.
35 (57 5 5 (1 12 8 2 Ans.
35
—
22
20
—
35) 445 (12s.
35
—
95
70
—
25
12
—
35) 305 (8d.
280
—
25
4
—
35) 100 (2qrs.
70
—
30

In 57 I find 35 once and 22 over. I then reduce 22 to shillings, adding the 5s. and in the sum 445s. I find 35 12 times and 25 over. I then reduce 25 to pence, adding 5d. and in the sum 305 I find 35 8 times and 25 over. Again, I reduce 25 to farthings, and divide by 35, and the quotient is 2qrs. and 30 remains, which is $\frac{30}{35}=\frac{6}{7}$ of another farthing.

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* The division of numbers of different denominations, or compound numbers, depends upon the same principles as Simple Division. This must be sufficiently obvious, when each of the several parts of the dividend can be divided without a remainder. And when there are remainders, the truth of the rule will

5. If 20 cwt. of tobacco cost £120 10s. what is it per cwt? Ans. £6 0s. 6d.

6. If 1 cwt. of tea cost £18 18s. what is it per pound? Ans. 3s. 4½d.

7. If 4 men spend at a tavern £2 16s. 4d. what must each pay? Ans. 14s. 1d.

8. If 12 silver cups weigh 13lb. 1oz. 2pwt. 10grs. what is the weight of each cup? Ans. 1lb. 1oz. 1pwt. 20⅚ grs.

9. If 24 lambs cost 30 dollars, what are they a piece? Ans. $1 25cts.

10. If 12 men draw a prize of 18000 dollars, what is each man's share? Ans. $1500.

11. If 147 bushels of wheat cost £47 12s. 6d. what is it per bushel? Ans. 6s. 5¾d.

12. If 196lb. of cotton cost £6 10s. 8d. what is it per pound? Ans. 8d.

QUESTIONS.

1. What is Compound Division?
2. How do you place the numbers?
3. How do you proceed in dividing?
4. What is to be done when there is a remainder after dividing any denomination?
5. What is the method of proof?

Miscellaneous Examples.

1. What is the difference between six dozen dozen and half a dozen dozen? Ans. 792.

2. What is the difference between half a solid foot and a solid half foot? Ans. 648 inches.

3. A note was on interest from March 20, 1819, till Jan. 26, 1824; what was the length of time? Ans. 4y. 10mo. 6ds.

years.	mo.	days.
1824	0	26
1819	2	20
4	10	6

In operations of this kind, a month is considered 30 days, and a year 12 months. This, though not perfectly correct, will be found to be a good practical method of ascertaining the time in computing interest.

4. How long from June 7, 1814, to August 3, 1823? Ans. 9ys. 1mo. 26ds.

5. Divide 5 guineas among 8 men—give A 8d. more than B, and B 8d. more than C, &c. what does H receive? Ans. 15s. 2d. H's share.

---

appear equally plain, by preparing the dividend in such a manner, before dividing, that the several parts may be divided without a remainder. If you would divide £3 13s. 8d. by 2, first make all the parts divisible by 2; thus £3 13s. 8d.=£2 32s. 20d. These parts divided successively by 2, give £1 16s. 10d. the same as by the rule.

In Compound Division there are usually given a variety of cases; but it was thought better to give one general rule which would answer every purpose without unnecessarily incumbering the memory of the scholar. After becoming familiarly acquainted with the rule here given, the several contractions will readily suggest themselves in practice from what has been taught in Simple Division.

6. A horse is valued by A at $60, by B at $69 50, and by C at $72 25, what is the average judgment?

A. 1	- -	$60
B. 1	- -	69 50
C. 1	- -	72 25
3		3) 201 75
		$67 25 Ans.

The average in this and similar cases, is found by dividing the sum of the several judgments by the number of appraisers.

7. M, N, O, and P, appraised a ship as follows, viz. M at $6700, N at $9000, O at $8750, and P at $7380; what is the average judgment? Ans. $7957 50.

8. A and B wishing to swap horses, and disagreeing as to the conditions, referred the matter to 3 disinterested persons, X, Y and Z, whose judgments were as follows, viz. X said A should pay B $8; and Y said A should pay B $6; but Z said B should pay A $5; what is the average judgment? Ans. A must pay B $3.

	A	B	
X 1.	$0	$8	14 B
Y 1.	0	6	5 A
Z 1.	5	0	—
			3) 9 (3 Ans.
Referees. 3	5	14	

In the exchange of articles, where the judgment of the referees is partly on one side of the equality between them, and partly on the other, subtract one side from the other, and divide the remainder by the number of referees for the average judgment.

9. C and D wishing to swap farms, referred the subject to O, P, Q and R, and agreed to abide their judgment, which was as follows, viz. O said C should pay D $70; P said C should pay D $100; and Q said C should pay D $55; but R said D should pay C $25; how was the matter settled? Ans. C pays D $50.

10. What is the weight of 4hhd. of sugar, each weighing 7cwt. 4qrs. 19lb.? Ans. 31cwt. 2qrs. 20lb.

11. Three men and 2 boys hoed 30000 hills of corn, and each man hoed 2 hills while a boy hoed one; how many hills were hoed by each man, and how many by each boy?

Ans. Each man hoed 7500 and each boy 3750 hills.

$3\times2+2=8$ Divisor.

12. If $911.555 be divided among 5 men and 4 women, what is each man and woman's share?

Ans. { $65.111=1 woman's share.
$130.222=1 man's share.

13. Two places differ in longitude 31° 37′ 3″ what is their difference in reckoning time, allowing 15° to make an hour?

Ans. 2h. 6′ 3″.

14. How much wood in a load 6ft. 7′ long, 3ft. 5′ high and 3ft. 8′ wide? Ans. 82ft. 5′ 8″ 4‴.

15. I bought a load of wood 8ft. long, 3ft. wide and 2ft. 8′ high, how much did it cost at the rate of $1.75 per cord?

Ans. 87½ cents.

SECTION III.

Fractions.*

Fractions are parts of a unit.

Fractions are of two kinds, *Vulgar* and *Decimal*, which differ in the manner of expression and modes of operation.

A Vulgar Fraction is expressed by two numbers, written one over the other with a line between; as $\frac{1}{2}$, $\frac{1}{8}$, $\frac{6}{7}$ and $\frac{15}{84}$.

The number below the line is called the *denominator*, and expresses the number of parts into which the unit is divided.

The number above the line is called the *numerator*, and shows how many of those parts are contained in the fraction; thus, the meaning of the expression, $\frac{3}{8}$ of a bushel, is, that a bushel is divided into 8 equal parts, and that 3 of those parts are taken.

A *Decimal Fraction* is expressed by one number, which is distinguished from a whole number by a period at the left hand, called the *separatrix*, as .5, .45, the first denoting 5 tenth parts, and the second 45 hundredth parts.

A Decimal may be changed into a Vulgar Fraction by drawing a line under it, and writing under the line as many cyphers as there are figures in the decimal, with a 1 at the left hand. Thus .5 is $\frac{5}{10}$, .45 is $\frac{45}{100}$ and .005 is $\frac{5}{1000}$.

1. *VULGAR FRACTIONS.*

Vulgar Fractions are those whose numerators and denominators are both expressed.

A *proper fraction* is one whose numerator is less than its denominator; as $\frac{1}{3}$, $\frac{4}{5}$, $\frac{3}{4}$, &c.

An *improper fraction* is one whose numerator is greater than its denominator; as $\frac{3}{2}$, $\frac{8}{3}$, &c.

A *compound fraction* is a fraction of a fraction, as $\frac{1}{3}$ of $\frac{3}{4}$, &c.

A *mixed number* consists of a whole number and a fraction, as $12\frac{1}{4}$, $6\frac{1}{8}$, &c.

A whole number is changed into an improper fraction by writing 1 under it, with a line between, as $\frac{4}{1}$, $\frac{6}{1}$, &c.

The common measure of two, or more numbers, is a number which will divide each of them without a remainder.

The greatest common measure of two, or more numbers, is the greatest number which will divide those numbers severally without a remainder.

The common multiple of two, or more numbers, is a number which may be divided by each of those numbers without a remainder.

---

* Fractions comes from the Latin word, *Frango*, to break, because the unit is considered as broken into several equal parts. Vulgar and Decimal Fractions differ in this; the denominator of the former may be any number whatever, but the denominator of the latter, when expressed, is always 10, 100, 1000, or 1 with as many ciphers annexed as there are figures in the decimal.

The least common multiple is the least number, which can be so divided without a remainder.

A prime number is one which can be measured only by itself or by a unit.

A perfect number is one which is equal to the sum of all its aliquot parts.*

1. *REDUCTION OF VULGAR FRACTIONS.*

Reduction of Vulgar Fractions, is changing them from one form into another without altering their value.

Case I.

To find the greatest common measure of two, or more numbers.

Rule.

1. If two numbers only be given, divide the greater by the less, this divisor by the remainder, and so on till nothing remains, always dividing the last divisor by the last remainder; then will the last divisor be the common measure required.

2. If there be more than two numbers given, find the greatest common measure of two of them; then of that common measure and one of the others, and so on through all the numbers; the greatest common measure last found will be the answer.

Examples.

1. What is the greatest common measure of 580, 320 and 45?

```
320)580(1            20)45(2
    320                 40
    ---                 --
    260)320(1      Ans. 5)20(4
        260               20
        ---
         60)260(4    Hence 5 is
            240      the great-
            ---      est number
Com. meas. } 20)60(3 which will
of 580 & 320}   60   divide 580,
                --   320 and 45
                     without a
                     remainder.
```

2. What is the greatest common measure of 918, 1998 and 522? Ans. 18.

3. What is the greatest common measure of 612 and 540? Ans. 36.

4. What is the greatest common measure of 1152 and 1080? Ans. 72.

* The aliquot part of any number is such a part of it, as, being taken a certain number of times, exactly makes that number. The smallest perfect number is 6. Its aliquot parts are 3, 2 and 1, and $3+2+1=6$. The next is 28, the next 496, and the next 8128. Only ten perfect numbers are yet known.

Case II.

To find the least common multiple of two, or more, numbers.

Rule.

1. Arrange the numbers in a line, and divide by any number that will divide two, or more, of the given numbers without a remainder, and set the quotients together with the undivided numbers in a line below.

2. Divide the second line as before, and so on till there are no two numbers remaining that can be thus divided; then will the continued product of the several divisors, and the figures in the last line, be the multiple required.

Examples.

1. What is the least common multiple of 3, 5, 8 and 10?

$$
\begin{array}{r|l}
5 & 3\ \ 5\ \ 8\ \ 10 \\ \hline
2 & 3\ \ 1\ \ 8\ \ 2 \\ \hline
 & 3\ \ 1\ \ 4\ \ 1
\end{array}
$$

$5\times2\times3\times4=120$ Ans.

Five and 10 divided by 5 the quotients are 1 and 2. with which 3 and 8 are bro't. down. Again 8 and 2 divided by 2, give 4 and 1 with which 3 and 1 are brought down. Then the product of 5, 2, 3 and 4 is the multiple required.

2. What is the least common multiple of 3, 4, 8 and 12?
Ans. 24.

3. What is the least number which can be divided by 6, 10, 16 and 20 without a remainder?
Ans. 240.

4. Supposing 12 clocks to be set a-going together, the first of which strikes at the end of every hour, the second at end of every second hour, the third at the end of every third hour, and so on to the 12th which strikes at the end of every 12 hours; how long before they will all strike together? Ans. 27720 hours.

Case III.

To reduce fractions to their lowest terms.

Rule.*

Divide both the terms of the fraction by their greatest common measure, and the quotient will be the fraction required.

---

* Dividing both terms of a fraction by the same number does not at all alter its value. If the greatest common measure of a fraction be 1, the fraction is already in its lowest terms.

Examples.

1. Reduce $\frac{48}{272}$ to its lowest terms.

48)272(5
  240
  —— Thus 16)$\frac{48}{272}=\frac{3}{17}$ An.
  32)48(1
     32
     —
Gr.com.mea.16)32(2
                 32

2. Reduce $\frac{57}{456}$ to its lowest terms. Ans. $\frac{1}{8}$.

3. Reduce $\frac{144}{240}$ to its lowest terms. Ans. $\frac{3}{5}$.

4. Reduce $\frac{1429}{2858}$ to its lowest terms. Ans. $\frac{1}{2}$.

Case IV.

To reduce a mixed number to its equivalent improper fraction.

Rule.*

Multiply the whole number by the denominator of the fraction, and add the numerator to the product; this sum written over the denominator will be the fraction required.

Examples.

1. Reduce $8\frac{2}{3}$ to an improper fraction.

8
3
—
24   then $\frac{26}{3}$ Ans.
 2
—
26

2. Reduce $27\frac{2}{9}$ to an improper fraction. Ans. $\frac{245}{9}$

3. Reduce $35\frac{4}{5}$ to an improper fraction. Ans. $\frac{179}{5}$

4. Reduce $100\frac{19}{59}$ to an improper fraction. Ans. $\frac{5919}{59}$

5. Reduce $36\frac{5}{8}$ to an improper fraction. Ans. $\frac{293}{8}$

Case V.

To reduce an improper fraction to its equivalent whole or mixed number.

Rule.†

Divide the numerator by the denominator and the quotient will be the whole number, and the remainder, if any, will be the numerator to the given denominator.

---

*All fractions represent a division of the numerator by the denominator, which taken together are proper and adequate expressions for the quotient. Thus 2 divided by 3, is 2·3; whence the reason of the rule is manifest; for if a quantity be multiplied and divided by the same number, it evidently remains the same. A whole number may be changed into an equivalent fraction with a given denominator, by multiplying the whole number by the denominator and writing the product over said denominator.

† This rule is evidently the reverse of the preceding, and is the same as Simple Division.

Examples.

1. Reduce $\frac{76}{3}$ to its equivalent whole or mixed number.

$$3\,)\,76\,(\,25\tfrac{1}{3}\text{ Ans.}$$

```
6
—
16
15
—
 1
```

2. Reduce $\frac{56}{8}$ to its equivalent whole number. Ans. 7.

3. Reduce $\frac{981}{16}$ to its equivalent mixed number. Ans. $61\frac{5}{16}$.

Case VI.

To reduce a compound fraction to an equivalent single one.

Rule.* Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator; then reduce this new fraction to its lowest terms.

Examples.

1. Reduce $\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{5}{8}$ to a single fraction.

$$\frac{1\times3\times5}{3\times5\times8}=\frac{15}{120}=\frac{1}{8}\text{ Ans.}$$

2. Reduce $\frac{3}{4}$ of $\frac{1}{3}$ of $\frac{5}{6}$ of $\frac{4}{11}$ to a single fraction. Ans. $\frac{5}{66}$.

Case VII.

To reduce fractions of different denominators to equivalent fractions having a common denominator.

Rule.† Multiply each numerator into all the denominators except its own for a new numerator, and all the denominators together for a common denominator.

Examples.

1. Reduce $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$ to a common denominator.

$1\times3\times4=12$ new num. for $\frac{1}{2}$
$2\times2\times4=16$ " " $\frac{2}{3}$
$3\times2\times3=18$ " " $\frac{3}{4}$
$2\times3\times4=24$ common denom.
Thus $\frac{12}{24}$, $\frac{16}{24}$ and $\frac{18}{24}$ Ans.

2. Reduce $\frac{1}{2}$, $\frac{1}{5}$ and $\frac{4}{7}$ to a common denominator. Ans. $\frac{35}{70}$, $\frac{14}{70}$, $\frac{40}{70}$.

3. Reduce $\frac{11}{13}$, $\frac{3}{4}$ of $1\frac{1}{4}$, $\frac{9}{11}$ and $\frac{5}{7}$ to a common denominator.
Ans. $\frac{13552}{16016}$, $\frac{15015}{16016}$, $\frac{13104}{16016}$, $\frac{11446}{16016}$

4. Reduce $\frac{1}{2}$, $\frac{4}{3}$ and $\frac{5}{6}$ to a common denominator. Ans. $\frac{18}{36}$, $\frac{48}{36}$, $\frac{30}{36}$.

---

* If part of the compound fraction be a whole or mixed number, it must be reduced to an improper fraction. If any denominator of a compound fraction be equal to a numerator of the same, both may be expunged, and the other numbers, multiplied as by the rule, will produce the fraction required in lower terms.

† By examining the operation it will be seen that the numerator and denominator of every fraction are multiplied by the very same numbers, and consequently their values are not altered.

Case VIII.

To reduce fractions of different denominators to equivalent fractions having the least common denominator.

RULE 1.*—Find the least common multiple of all the denominators of the given fractions, and it will be the common denominator required.

2. Divide the common denominator by the denominator of each fraction, and multiply the quotient by the numerator, the products will be the numerators required.

Examples.

1. Reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{5}{6}$ to the least common denominator.

```
2)2 3 6     6÷2=3 and 3×1=3 )
-------     6÷3=2     2×1=2 } New numerators.
3)1 3 3     6÷6=1     1×5=5 )
  -----
  1 1 1   2×3=6 least com. mult. then 3/6, 2/6, 5/6 Ans.
```

2. Reduce $\frac{7}{9}$ and $\frac{11}{18}$ to the least common denominator. Ans. $\frac{14}{18}$, $\frac{11}{18}$.

3. Reduce $\frac{1}{2}$, $\frac{5}{8}$, and $\frac{3}{8}$ to the least common denominator. Ans. $\frac{4}{8}$, $\frac{5}{8}$, $\frac{3}{8}$.

Case IX.

To find the value of a fraction in known parts of an integer.

RULE.—Multiply the numerator by the parts of the next inferior denomination, and divide the product by the denominator; if any thing remain, multiply it by the next inferior denomination, and divide by the denominator as before, and so on as far as necessary; the quotients will be the answer required.

Examples.

1. What is the value of $\frac{3}{8}$ of a pound?

```
 3
20
--
8)60(7s.      Ans. 7s. 6d.
  56
  --
   4
  12
  --
8)48(6d.
```

2. What is the value of $\frac{3}{12}$ of a pound? Ans. 5s.

3. What is the value of $\frac{3}{7}$ of a day? Ans. 10h. 17m. $8\frac{4}{7}$s.

4. What is the value of $\frac{5}{6}$ of a mile? Ans. 6fur. 26rds. 3yds. 2ft.

---

* The common denominator is a multiple of all the denominators, and consequently will divide by any of them: therefore, proper parts may betaken for all the numerators as required.

Case X.

To reduce a fraction of one denomination to that of another, retaining the same value.

Rule.—Make a compound fraction of it, and reduce it to a single one.

Examples.

1. Reduce $\frac{5}{6}$ of a penny to the fraction of a pound.
$\frac{5}{6}$ of $\frac{1}{12}$ of $\frac{1}{20}$, compound fraction.
Then $\frac{5}{6}\times\frac{1}{12}\times\frac{1}{20}=\frac{5}{1440}=\frac{1}{288}$, Ans.

2. Reduce $\frac{6}{7}$ of a pound to the fraction of a cwt. Ans. $\frac{3}{392}$.

3. Reduce 3s. 6d. to the fraction of a pound. Ans. $\frac{7}{40}$.

4. Reduce $\frac{4}{5}$ of a pound to the fraction of a guinea. Ans. $\frac{4}{7}$.

5. Reduce $\frac{1}{18}$ of a pound to the fraction of a penny.
$\frac{1}{18}$ of $\frac{20}{1}$ of $\frac{12}{1}$ comp. fraction.
Then $\frac{1}{18}\times\frac{20}{1}\times\frac{12}{1}=\frac{240}{18}=\frac{40}{3}$ Ans.

6. Reduce $\frac{3}{13}$ of a month to the fraction of a day. Ans. $\frac{84}{13}$.

7. Reduce $\frac{1}{126}$ cwt. to the fraction of a pound. Ans. $\frac{8}{9}$.

8. Reduce $\frac{7}{1920}$ lb. Troy, to the fraction of a pwt. Ans. $\frac{7}{8}$.

2. *ADDITION OF VULGAR FRACTIONS.*

Rule.—Reduce compound fractions to single ones; mixed numbers to improper fractions, fractions of different integers to those of the same, and all of them to a common denominator; then the sum of the numerators, written over the common denominator, will be the sum of the fractions required.

Examples.

1. Add $\frac{5}{8}$, $7\frac{1}{2}$, and $\frac{1}{3}$ of $\frac{3}{4}$ together.
First $7\frac{1}{2}=\frac{15}{2}$, and $\frac{1}{3}$ of $\frac{3}{4}=\frac{3}{12}$.
Then $\frac{5}{8}$, $\frac{15}{2}$, and $\frac{3}{12}$ are the fractions.

$$\begin{array}{lr} 5\times2\times12= & 120 \\ 15\times8\times12= & 1440 \\ 3\times8\times\ 2= & 48 \\ \hline & 1608 \end{array}$$

$$\frac{1608}{192}=8\tfrac{72}{192}=8\tfrac{3}{8}\ \text{Ans.}$$

$$8\times2\times12 \quad 192$$

2. What is the sum of $\frac{1}{13}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ of an hour? Ans. 2d. $14\frac{1}{2}$h.

3. What is the sum of $\frac{9}{10}$ of $6\frac{7}{8}$, $\frac{4}{7}$ of $\frac{1}{2}$, and $7\frac{1}{2}$? Ans. $15\frac{109}{112}$.

4. Add $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot, and $\frac{3}{8}$ of a mile.
Ans. 660 yds. 2 ft. 9 in.

---

* By reducing fractions to a common denominator, they are made to express similar parts of the same unit, and as each numerator shows how many of those parts are signified by the fraction, the sum or difference of the numerators written over the common denominator, is evidently the sum or difference of the fractions.

3. *SUBTRACTION OF VULGAR FRACTIONS.*

Rule.—Prepare the fractions as for addition, and the difference of the numerators written over the common denominator will be the difference of the fractions required.

Examples.

1. From $\frac{2}{3}$ take $\frac{2}{9}$ of $\frac{3}{7}$.
$\frac{2}{9}$ of $\frac{3}{7}=\frac{6}{63}=\frac{2}{21}$ and $\frac{2}{3}-\dagger\frac{2}{21}=\frac{42}{63}-\frac{6}{63}=\frac{14}{21}-\frac{2}{21}=\frac{12}{21}=\frac{4}{7}$ Ans.

2. From $96\frac{1}{3}$ take $14\frac{3}{7}$.
Ans. $81\frac{19}{21}$.

3. From $\frac{49}{50}$ take $\frac{5}{9}$.
Ans. $\frac{191}{450}$.

4. From 7 weeks take $9\frac{7}{10}$ days.
Ans. 5 w. 4 d. 7 h. 12 m.

5. From $\frac{1}{2}$£ take $\frac{3}{4}$s.
Ans. 9s. 3d.

6. From $14\frac{1}{4}$ take $\frac{2}{3}$ of 19.
Ans. $1\frac{7}{12}$.

4. *MULTIPLICATION OF VULGAR FRACTIONS.*

Rule.—Reduce compound fractions to single ones, and mixed numbers to improper fractions; then multiply the numerators together for the numerator, and the denominators together for the denominator of the fraction required.

Examples.

1. Multiply $4\frac{1}{2}$, $\frac{3}{4}$ of $\frac{1}{7}$, and $18\frac{4}{5}$ continually together.
$4\frac{1}{2}=\frac{9}{2}$. $\frac{3}{4}$ of $\frac{1}{7}=\frac{3}{28}$, and $18\frac{4}{5}=\frac{94}{5}$.
Then $\frac{9}{2}\times\frac{3}{28}\times\frac{94}{5}=\frac{2538}{280}=9\frac{18}{280}=9\frac{9}{140}$ Ans.

2. Multiply $5\frac{1}{4}$ by $\frac{1}{6}$. Ans. $\frac{7}{8}$.

3. Multiply $4\frac{1}{2}$ by $\frac{1}{8}$. Ans. $\frac{9}{16}$.

4. Multiply $\frac{1}{9}$ of $\frac{3}{5}$ by $\frac{5}{8}$ of $3\frac{3}{7}$.
Ans. $\frac{1}{7}$.

5. Multiply $\frac{4}{17}$ by $\frac{1}{4}$. Ans. $\frac{1}{17}$.

6. Multiply $\frac{1}{3}$ of 5 by $\frac{3}{4}$ of 4.
Ans. 5.

7. Multiply $\frac{1}{2}$ by $\frac{1}{3}$. Ans. $\frac{1}{6}$.

5. *DIVISION OF VULGAR FRACTIONS.*

Rule.—Prepare the fractions as for Multiplication, then invert the divisor, and proceed exactly as in Multiplication.

---

† Fractions are sometimes most conveniently brought to a common denominator by Multiplication or Division. In the first example $\frac{2}{3}$ is brought to a common denominator with $\frac{2}{21}$ by multiplying both its terms by 7.

Examples.

1. Divide $9\frac{1}{6}$ by $\frac{1}{2}$ of 7.

$9\frac{1}{6}=\frac{55}{6}$ dividend.

$\frac{1}{2}$ of $\frac{7}{1}=\frac{7}{2}$ divisor. Then,

$\frac{55}{6}\times\frac{2}{7}=\frac{110}{42}=2\frac{26}{42}=2\frac{13}{21}$ Quot.

2. Divide $\frac{4}{7}$ by $\frac{1}{2}$. Ans. $\frac{8}{7}=1\frac{1}{7}$.

3. Divide $\frac{7}{9}$ by 4. Ans. $\frac{7}{36}$.

4. Divide $\frac{1}{2}$ of $\frac{1}{4}$ by $\frac{1}{3}$ of $\frac{3}{4}$. Ans. $\frac{1}{2}$.

5. Divide $5\frac{1}{8}$ by $7\frac{3}{4}$. Ans. $\frac{41}{62}$.

6. Divide $\frac{1}{3}$ by $\frac{6}{8}$. Ans. $\frac{4}{9}$.

QUESTIONS.

1. What are Fractions?
2. Of how many kinds are Fractions?
3. Wherein do they differ?
4. How is a Vulgar Fraction expressed?
5. What is the number below the line called?
6. What does it express?
7. What is the number above the line called?
8. What does that show?
9. How is a Decimal Fraction expressed?
10. How is it distinguished from a whole number?
11. How may a Decimal be changed into a Vulgar Fraction?
12. What is a Vulgar Fraction?
13. What is a proper fraction?
14. What is an improper fraction?
15. What is a compound fraction?
16. What is a mixed number?
17. What is the common measure of two or more numbers?
18. What is the greatest common measure of two or more numbers?
19. What is the common multiple of two or more numbers?
20. What is the least common multiple?
21. What is a prime number?
22. What is a perfect number?
23. What is Reduction of Vulgar Fractions?
24. How do you find the greatest common measure of two or more numbers?
25. How do you find the least common multiple?
26. How are fractions reduced to their lowest terms?
27. How are mixed numbers reduced to improper fractions?
28. How is an improper fraction reduced to whole or mixed numbers?
29. How is a compound fraction reduced to a single one?
30. How are fractions reduced to a common denominator?
31. How to the least common denominator?
32. What is the rule for the Addition of Vulgar Fractions?
33. ——— for the Subtraction?
34. ——— for the Multiplication?
35. ——— for the Division?

2. Decimal Fractions.

A Decimal Fraction is one whose numerator only is expressed. Were the denominator to be written, it would, in all cases, be 10, 100, or 1, with as many ciphers annexed as there are figures in the decimal.

When there are whole numbers and decimals in the same sum, it is called a *mixed number*, as, 16.44, which is read sixteen and forty-four hundredths.

In decimals, unity is considered a fixed point, each way from which the value of the numbers varies in a ten-fold proportion, increasing towards the left hand, and decreasing towards the right, as in the following

TABLE.

100 Millions.	*10 Millions.*	*Millions.*	*100 Thousands.*	*10 Thousands.*	*Thousands.*	*Hundreds.*	*Tens.*	*Units.*	*Tenth parts.*	*Hundredths.*	*Thousandths.*	*10 Thousandths.*	*100 Thousandths.*	*Millionths.*	*10 Millionths.*	*100 Millionths.*
9	8	7	6	5	4	3	2	1 .	2	3	4	5	6	7	8	9

Ciphers in the right hand of decimals do not alter their value, but on the left hand, decrease their value in a ten-fold proportion. Thus, .5 .50 and .500 express the same value, viz. ½, and are read 5 tenths, 50 hundredths, and 500 thousandths; but .5 .05 and .005 decrease in value in a ten fold proportion, and are read 5 tenths, 5 hundredths, and 5 thousandths.

In order to make the scholar familiar with the notation of decimals, he is requested to write out the following

Examples.

1. Express the decimal .36 in words.
2. Express .03 in words.
3. Express .1002 in words.
4. Express 27.27 in words.
5. Express 34.14 in words.

1. Write forty-three thousandths in characters.
2. Write 60 and nine hundred thousandths in characters.
3. Write one hundred and four thousandths.

1. *ADDITION OF DECIMALS.*

Rule.

Place the numbers under each other according to the value of their places, and add as in whole numbers. Point off as many decimal places from the sum as are equal to the greatest number of decimal places in either of the given numbers.*

---

* When the numbers are all written properly, and the amount properly pointed, the separatrices, or decimal points, will all stand in a column, or directly

Examples.

1. What is the sum of 25.4 rods, 16.05 rods, 8.842 rods and 46.004, when added together?

25.4	Here it will be seen that the decimal points all fall in the same column, that the decimals are arranged towards the right hand from this column, and the whole numbers towards the left, and that the number of decimals in the sum is equal to the greatest given number of decimals.
16.05	
8.842	
46.004	
96.296 Ans.	

2. What is the sum of 312.984, 21.3918, 2700.42, 3.153, 27.2, and 581.06? Ans. 3646.2088.

3. What is the sum of thirty-seven and eight hundred twenty-one thousandths; five hundred and forty-six and thirty-five hundredths; eight and four tenths, and thirty seven and three hundred twenty-five thousandths? Ans. 629.896.

4. What is the sum of .014, .9816, .32, .15914, .72913 and .0047? Ans. 2.20857

5. What is the sum of six thousand and six thousandths; five hundred and five hundredths, and forty and four tenths? Ans. 6540.456

2. *SUBTRACTION OF DECIMALS.*

Rule.—Place the less number under the greater, with one of the decimal points directly under the other; then subtract as in whole numbers, and point off as in addition.

Examples.

1. From 468.742 take 76.4815

    468.742
     76.4815

    Rem. 392.2605

2. From 2.73 take 1.9183
    Rem. 0.8115

3. From 428, subtract 14.76
    Ans. 413.24

4. From .9173 subtract .2138
    Ans. .7035

5. From 742 subtract 195.127
    Ans. 546.873

6. From 9.005 subtract 8.728
    Ans. 0.277

---

over one another. All the difficulty in Decimal Fractions is in placing the numbers, and pointing off the decimals. In other respects, they are managed precisely as whole numbers. The scholar should endeavor to become familiar with the management of Decimals, as to us they form one of the most useful parts of Arithmetick. Our lawful mode of reckoning money is purely decimal.

3. MULTIPLICATION OF DECIMALS.

RULE.*—Write the multiplier under the multiplicand; then multiply as in whole numbers, and from the product point off as many places for decimals, as there are decimal places in both the factors. If there be not so many figures in the product as there ought to be decimals, supply the deficiency by prefixing ciphers.

Examples.

1. Multiply 25.63 by 2,4

```
 25.63
   2.4
 -----
 10252
 5126
 -----
 61.512
```

Here because there are three decimal places in both factors, I point off three places in the product.

2. Multiply 25.238 by 12.17 Ans. 307.14646

3. Multiply .026 by .003

```
         .026
         .003
       -------
Pro. .000078
```

Here because there are six decimal places in both factors, I make up the deficiency of the product by placing four ciphers at the left hand of 78.

4. Multiply 17.6 by .75 Ans. 13,2

4. DIVISION OF DECIMALS.

RULE.†—Divide as in whole numbers, and point off so many places for decimals in the quotient as the decimal places in the dividend exceed those in the divisor. If there are not so many figures in the quotient as the number of decimals required, supply the defect by prefixing ciphers. If the decimal places in the divisor exceed those in the dividend, make them equal by annexing ciphers to the latter. When there is a remainder, by annexing ciphers, more decimal places may be obtained in the quotient.

---

* The truth of this rule will appear by considering that .026 and .003 are equivalent to $\frac{26}{1000}$ and $\frac{3}{1000}$; whence $\frac{26}{1000}\times\frac{3}{1000}=\frac{78}{1000000}=$ .000078 by the nature of notation; that is, the decimal consists of as many places as there are ciphers in the denominator, and when the product falls short of this number, the deficiency must be made up by ciphers on the left hand. There are usually given several methods of contraction under this rule; but they are of no essential service, and might perplex the young scholar. It may not be amiss, however, to observe that in dividing by 10, 100, or 1 with any number of ciphers, we have only to remove the separatrix as many places towards the right hand as there are ciphers in the multiplier; thus 2.71 multiplied by 10 is 27.1; by 100, it is 271, &c.

† The reason of the rule for pointing off the decimal places in the quotient will appear obvious by considering that the divisor and quotient are two factors whose product is the dividend, and that the decimal places in both the factors are equal to the decimal places in their product, as was shown in Multiplication of decimals.

Examples.

1. Divide 487.653 by 24·21

```
24.21)487 653(20.14+
      4842
      ----
        3453
        2421
        ----
        10320
         9684
         ----
          636
```

Here are four decimals in the dividend, (counting the cipher added to the remainder after bringing down all the figures in the dividend) and only two in the divisor, therefore there must be two decimal places pointed off in the quotient, that the decimal places to the quotient and divisor counted together may equal those in the dividend. The sign + plus after the quotient, shows that more decimals may be procured by annexing ciphers to the remainder.

2. Divide 7.02 by .18
   Ans. 39.

3. Divide .0081892 by .347
   Ans. 0236

The Scholar is requested to point the two following examples.

4. Divide 4263 by 2.5
   Ans. 17052

5. Divide 4.2 by 36.
   Ans. 116+

RECIPROCALS.

The *Reciprocal* of a given number is one, which, multiplied by the given number, gives a unit for the product; thus .2 is reciprocal of 5, because $5\times.2=1$.

If the given number be a multiplier, its reciprocal may be employed as a divisor of the multiplicand, and the quotient will be equal to the product of the multiplicand by the multiplier; but if the given number be a divisor, its reciprocal may be employed as a multiplier of the dividend, and the product will be equal to the quotient of the dividend by the divisor. *Examples.*—1. Multiply 7 by 5. $7\times5=35$, and $7\div.2=35$. 2. Divide 7 by 5. $7\div5=1.4$, and $7\times.2=1.4$.

Problem.

To find the reciprocal of any number.

Rule.—Divide a unit by the given number, and the quotient will be its reciprocal.

Examples.

1. What is the reciprocal of 125?

```
125)1.000(.008 Ans.
    1000
    ----
```

$125\times.008=1$. proof.

2. What is the reciprocal of .4?
   Ans. 2.5

3. By what number shall I multiply 240, that the product may be equal to the quotient of 240 divided by 25?

```
25)1.00(.04 recip. Ans.
   100
   ---
```

$\left.\begin{array}{l}340\times.04=9.6\\240\div25=9.6\end{array}\right\}$ proof.

5. REDUCTION OF DECIMALS.

Case I.

To reduce Vulgar Fractions to Decimals.

RULE.—Annex a cipher to the numerator, and divide it by the denominator; annex a cipher to the remainder, and divide as before, and so on; the quotient will be the decimal required.*

Examples.

1. Reduce $\frac{3}{4}$ to a decimal.

```
4)3.0(.75 Ans.
  28
  --
   20
   20
   --
```

2. Reduce $\frac{1}{2}$ and $\frac{1}{4}$ to decimals. Ans. .5, .25

3. Reduce $\frac{1}{3}$ to a decimal. Ans. .333+

4. Reduce $\frac{11}{22}$ to a decimal. Ans. .5

5. Reduce $\frac{31}{44}$ to a decimal. Ans. .7045+

6. Reduce $\frac{13}{75}$ to a decimal. Ans. .173+

Case II.

To reduce numbers of different denominations to their equivalent decimal values.

RULE.*—Write the numbers perpendicularly under each other for dividends, proceeding orderly from the least to the greatest, and write on the left hand of each, for a divisor, the number which it takes of that to make one of the next higher denomination, and draw a line perpendicularly between the divisors and dividends.

Begin with the highest, and write the quotient of each division as decimal parts, on the right of the next dividend below. Continue thus to do till all the dividends are used, and the last quotient will be the decimal required.

Examples.

1. Reduce 12s. 9d. 3q. to the decimal of a pound.

4	3
12	9.75
20	12.8125
	.640625 dec. required.

The given numbers stand as integers. In the first place I annex 2 ciphers to 3, making it 3.00, and dividing it by 4, the quotient .75, the decimal of a penny, which I set against the 9d. then dividing by 12, I get .8125; and then by 20, I get .640625, the decimal of a pound.

---

* If the number of figures in the quotient be just equal to the number of ciphers annexed to the dividend, then the quotient is the true decimal; but if it be less, it must be made equal by placing ciphers at the left hand. In cases where the numerator is greater than the denominator, it is an improper fraction, and the quotient will be a whole or mixed number.

† The reason of this rule will appear by a little attention to the first example. Here 3 qrs. is $\frac{3}{4}$ of a penny, which, reduced to a decimal, is .75. Hence 9.75 $=9\frac{3}{4}$d. But 9.75 is $\frac{975}{100}$ of a penny $=\frac{975}{1200}$ of a shilling, which reduced to a decimal is .8125, and therefore 12s. $9\frac{3}{4}$d. may be expressed thus, 12.8125. In like manner, 12.8125s. is $\frac{128125}{10000}$ of a shilling $=\frac{128125}{200000}$ of a pound, which reduced to a decimal is .640625£ as found by the rule.

2. Reduce 10 oz. 18 pwt. 16 grs. to the decimal of a pound. Ans. .911111+

3. Reduce 3 qrs. to the decimal of a shilling. Ans. .0625.

4. Reduce 19s. 5½d. to the decimal of a pound. Ans. .971875

5. Reduce 2 qrs. 21 lb. 12 oz. 10 dr. to the decimal of a cwt. avoirdupois? Ans. .6945452

Case III.

To reduce shillings, pence and farthings to the decimal of a pound by inspection.

RULE.*—Write half the greatest even number of shillings for the first decimal figure, consider how many farthings there are in the given pence and farthings, and let these possess the second and third places; remembering to increase the second place by 5, if the shillings be an odd number, and the third place by 1, when the farthings exceed 12, and by 2 when they exceed 36.

Examples.

1. Reduce 13s. 10½d. to the decimal of a pound, by inspection.

6=½ of 12, the greatest even number of shillings.
5 for the odd shilling.
42=farthings in 10½d.
2 because the farthings exceed 36.

.694 the decimal required.

2. Find the decimal value of 15s. 8½d. by inspection. Ans. .785

3. Find the decimal value of 9½d. by inspection. Ans. .040

4. Find the decimal value of 6s. by inspection. Ans. .300

5. Find the decimal value of 1s. 10d. by inspection. Ans. .092

6. Find the decimal value of 16s. 4½d. by inspection. Ans. .819

* As shillings are so many 20ths of a pound, half the shillings are so many 10ths of a pound; therefore half the even number of shillings will occupy the place of *tenths* in the decimal. When there is an odd shilling, it is just equal ½ a tenth, or a $\frac{5}{100}$ part of a pound; it is therefore properly expressed by a 5 in the second decimal place. A pound is equal to 960 farthings; now had it happened that 1000 qrs. instead of 960, had made a pound, farthings would have been so many thousands of a pound, and might have been placed in the decimal as such. But 960 falls short of 1000 just $\frac{1}{24}$ part of itself; consequently, any number of farthings, increased by its $\frac{1}{24}$ part, will be an exact decimal expression for it. When the farthings are over 12, and less than 36, a $\frac{1}{24}$ is more than ½, and less than 1½, and therefore 1 must be added to give the nearest decimal in the third place; and when the farthings exceed 36, a $\frac{1}{24}$ part is more than 1½, and therefore 2 must be added. This gives the decimal sufficiently correct for common practice; but when greater exactness is required, a $\frac{1}{24}$ is to be found by division, and the decimal places increased.

Case IV.

To find the value of any given decimal in the terms of an integer.

Rule.—Multiply the decimal by that number which it takes of the next less to make one of the denomination in which the decimal is given, and cut off from the right hand as many places for a remainder as there are places in the given decimal. Proceed with the remainder in the same way, and so on through all the denominations; the numbers standing on the left of the parts cut off will form the answer.

Examples.

1. What is the value of .640625 of a pound?

```
  .640625
       20
 --------
12.812500
       12
 --------
 9.750000
        4
 --------
 3.000000
```

Ans. 12s. 9d. 3qrs.

The examples under this case are the examples under Case II. inverted, by which it will be seen that they reciprocally prove each other.

2. What is the value of .972916 of a pound?

Ans. 19s. 5d. 1qr.

3. What is the value of .911111+ of a pound troy?

Ans. 10oz. 18pwt. 15grs.*

4. What is the value of .0625 of a shilling? Ans. 3qrs.

Case V.

To find the value of any decimal of a pound by inspection.

Rule.†—Double the first figure in the decimal for shillings, and if the second figure be 5, or more than 5, reckon another shilling; then call the figures in the second and third places, (after 5, if contained in the second, is deducted,) so many farthings; abating 1 when they are above 12; and 2, when they are above 36; and the result is the answer.

---

* By comparing the answer to this example, with its correspondent sum in Case II. it will appear that a grain is lost. But it will be seen that the decimal which remains, approaches very nearly to another integer, and the loss is because the complete value of the decimal is not employed in the operation. It is usually best to take the lowest denomination to the nearest integer; that is, when the first figure of the decimal is more than 5, add 1 to the integer. In this way, the numbers in the example will agree with their correspondent numbers in Case II.

If, after having obtained the integers of the lowest denomination, a decimal remain, and it is required to find its value precisely, change it to a Vulgar Fraction, and reduce it to its lowest terms by dividing both parts of the fraction as long as you can find any number which will divide them both without a remainder. Supposing the decimal were .75, it is $\frac{75}{100}$. Now 75 and 100 may both be divided by 25; thus, $25)\frac{75}{100}=\frac{3}{4}$, and as there is no number which will divide 3 and 4 without a remainder, $\frac{3}{4}$ is the lowest, or most simple expression for $\frac{75}{100}$.

† This rule is the converse of that given under Case III. and the reason of it must be sufficiently obvious, from what was there said.

Examples.

1. Find the value of .785 of a pound by inspection.

14s. - double 7.
1s. - for 5 in the 2d place.
8d. 3qrs.= 35 qrs. abat. 5 from 8.
1qr. for excess of 12, abated.

15s.8d.2qrs. Ans.

2. Find the value of .875 of a pound by inspection.
Ans. 17s. 6d.

3. Find the value of .3 of a pound by inspection.
Ans. 6s.

4. Find the value of .040 of a pound by inspection.
Ans. 9½d.

5. Find the value of .092 of a pound by inspection.
Ans. 1s. 10d.

6. Find the value of .819 of a pound by inspection.
Ans. 16s. 4½d.

7. Find the value of .694 of a pound by inspection.
Ans. 13s. 10½d.

Application of the preceding Rules in Fractions.

1. What is the sum of 8½ rods, 12¼ rods, and 2¾ rods, when added together?

8.5
12.25
2.75

23.50 Ans.

The scholar should bear in mind that where these expressions, ¼, ½, and ¾ occur, he is to substitute for them their equivalent decimal values, which are respectively .25, .5 and .75.

2. From 15½ rods, take 3¾ rods. Ans. 11.75 rods.

3. Multiply 12¼ feet by 3¼ feet. Ans. 39.8125 feet.

4. How many feet in a board 9½ feet long, and 2¼ feet wide? Ans. 21.375 feet.

5. How much wood in a pile 20 feet long, 3½ feet wide, and 6¼ high?
Ans. 437.5 feet=3 cords, 53 feet, 864 inches.*

6. Into how many pieces ¼ of a foot long, may a pole 15.5 feet long, be cut? Ans. 62.

7. What is the value of .875 of a day? Ans. 21 hours.

8. How many feet of boards will cover the two sides of a barn, they being each 36½ feet long, and 14.2 feet high?
Ans. 1036.6 feet.

9. How many acres in a piece of land 74.8 rods long, and 45.6 broad?
Ans. 21 acres, 50.88 rods.

10. How many rods in a piece of land 16½ rods long, and 15½ wide?
Ans. 255.75 rods,=1 acre, 95¾ rods.

11. How many solid feet in a pile of timber 25 feet long, 14.2 feet wide and 14.2 feet high?
Ans. 5041 feet.

* The feet are brought into cords by dividing by 128, the number of feet in a cord. The .5 of a foot is reduced to inches by Case IV. Reduction of Decimals.

QUESTIONS.

1. What is a Decimal Fraction?
2. What would be its denominator were it expressed?
3. What is a whole number and decimal in the same sum called?
4. How does the value of the numbers vary from unity?
5. What effect have ciphers placed on the right and left hand of Decimals?
6. What is the rule for the addition of decimals?
7. For Subtraction?
8. For Multiplication?
9. For Division?
10. Repeat in short the method of pointing off in each of these rules.
11. How are Vulgar Fractions reduced to Decimals?
12. How are numbers of different denominations reduced to their equivalent decimal values?
13. How are shillings, pence, and farthings reduced to the decimal of a pound by inspection?
14. How is the value of a given decimal found in the terms of an integer?
15. How do you find the value of the decimal of a pound by inspection?

Federal Money.*

Federal Money is the established coin of the United States. Its denominations are in a decimal or ten-fold proportion, and were determined by act of Congress, August 8, 1786.

The different denominations in Federal Money are exhibited in the following

Table.

10 Mills, *m.*	make 1	Cent, *marked*	*ct.*
10 Cents	make 1	Dime,	"	*d.*
10 Dimes	make 1	Dollar,	"	$ or *doll.*
10 Dollars	make 1	Eagle.	"	*E.*

* For its simplicity and ease of reckoning, Federal Money is superior to any other, and it is fast supplanting, as it should do, the old method of computing by pounds, shillings, pence, and farthings. Could the same improvements be made in weights and measures, a competent knowledge of Arithmetick could be obtained with one half the labor which is now required, and the same computations could be made in half the time.

The standard for gold and silver is eleven parts fine, and one part alloy, or, as goldsmiths would say, 22 carats fine. A carat is $\frac{1}{24}$ part of any quantity, and when gold or silver is said to be 22 carats fine, it is to be understood, that were the whole mass divided into 24 parts, 22 of them would be pure gold or silver, and the other 2 alloy. Copper is commonly used as alloy in gold and silver, and is employed to render them more hard and durable. The weight of our coins are as follows: Eagle, 11 pwts. 6 grs. Half Eagle, 5 pwts. 15 grs. Dollar, 17 pwts. 7 grs. Half dollar, 8 pwts. 16 grs. and 100 cents weigh $2\frac{1}{4}$lb. avoirdupois. The denominations less than a dollar are expressive of their values; thus, *mill* is from the Latin *mille*, a thousand; for 1000 mills make 1 dollar; *cent* is from *centum*, a hundred, because 100 cents make 1 dollar, and *dime* is from the French, signifying the tenth part, because 10 dimes make 1 dollar. Uncoined gold, 22 carats fine, is worth at the mint, $209.77 per pound troy, and uncoined silver, of the same fineness, is worth $9.92 per pound troy.

The dollar is considered to be the *unit money*, and all denominations below are decimal parts of a dollar. Thus, 1 dime is .1, or $\frac{1}{10}$ of a dollar, 1 cent .01, or $\frac{1}{100}$ of a dollar, and 1 mill, .100 or $\frac{1}{1000}$ of a dollar. The place next to dollars on the left hand, is eagles. Any number of dollars, as 475, may either be read 475 dollars, or 47 eagles, 5 dollars; and the decimal parts of a dollar, as .865 may be read 8 dimes, 6 cents, 5 mills, or 86 cents, 5 mills, or 865 mills. Hence a sum expressed in Federal Money, is a mixed number in Decimal Fractions, and may be managed as such. Thus 25 eagles, 8 dollars, 4 dimes, 6 cents and 3 mills are written

Eagles.	Dollars.	Dimes.	Cents.	Mills.
2 5	8.	4	6	3

or

Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.
2	5	8.	4	6	3

The usual way of reading sums in Federal Money is by naming only three of the denominations, namely, dollars, cents and mills. In this way, the above sum would be read 258 dollars, 46 cents, and 3 mills.

The real coins in Federal Money are two of gold, *the Eagle* and *half eagle*, four of silver, the *dollar, half dollar, double dime*, and *dime*, and two of copper, the *cent* and *half cent*. The *mill* is only imaginary, there being no piece of money of that denomination.

Addition, Subtraction, Multiplication and *Division* of Federal Money, are performed by the rules already given for Addition, Subtraction, Multiplication and Division in Decimal Fractions, and to these the scholar is referred.

Examples.

1. What is the sum of 5 eagles, 6 dollars, 8 dimes, 5 cents, \$9.444, \$1.002, \$25.22 and .867 when added together?

```
E.D. d. ct.m.
5 6. 8 5
  9. 4 4 4
  1. 0 0 2
2 5. 2 2
     8 6 7
---------
9 3. 3 8 3
```

2. Suppose a man purchase 4 handkerchiefs at .62 cents each, 8 yds. ribbon at .17 cts. per yard, and 5 yds. of lace at .44cts. per yard; what is the amount of the purchase? Ans. \$6.04.

3. What will 156 yds. of cloth cost at \$1.67 per yard? Ans. \$260.52.

4. A man bought 2½yds. broadcloth for \$15.50, 6 yds. of silk, for \$5.75, 7 yds. of cambrick for \$5.25, and trimmings to the amount of \$4.22; what was the amount of the purchase? Ans. \$30.72.

5. What will 47yds. of cloth cost, at .22 cents per yard? Ans. \$10.34.

The scholar will bear in mind the rule for pointing in the Multiplication of decimals.

6. A man bought 24 yds. of cloth at \$1.50 per yard, and paid \$26.55, how much remains due? Ans. \$9.45.

7. If 24 lb. of tea cost \$7.97, what is it per pound? Ans. \$0.332.

8. If 125 bushels of wheat cost \$200.50, what is it per bushel? Ans. \$1.604.

9. If a man buy 154 bushels of wheat at \$0.95 per bushel, and sell it at \$1.125 per bushel, what is his gain? Ans. \$26.95.

10. If \$1268. be divided equally among 15 men, what sum does each receive? Ans. \$84.533.

When there is a remainder after dividing the dollars, the cents and mills are obtained by annexing ciphers.

11. Six men, in company, purchased 27 bushels of salt at \$1.67 per bushel; what did each man pay, and what was each man's share of the salt? Ans. \$7.515 and his share 4½ bushels.

12. A man dies leaving an estate of \$35000, the demands against the estate are \$1254.65; the remainder, after deducting a legacy of \$3075, is divided equally among his 6 sons; what is each son's share? Ans. \$5111.725.

QUESTIONS.

1. What is Federal Money?
2. In what proportion are its denominations?
3. How and when were they determined?
4. What is the table of Federal Money?
5. Which is the unit money?
6. What are the denominations below?
7. What part of a dollar does one dime express? —one cent? —one mill?
8. What is a sum expressed in Federal Money?
9. What denominations are generally mentioned in naming a sum in Federal Money?
10. What are the real coins in Federal Money?
11. What imaginary?
12. How are Addition, Subtraction, Multiplication and Division performed in Federal Money?

REDUCTION OF FEDERAL MONEY.

Case I.

*To reduce New-England, &c. and New-York, &c. currencies to Federal Money.**

RULE.—If there be shillings, pence and farthings in the given sum, reduce them to the decimal of a pound by inspection, (see Case III. Reduction of Decimals,) and place it at the right hand of the pounds. Divide the given sum thus prepared, by .3, if New-England, and by .4 if New-York currency, and the quotient, pointed according to the rule for the division of decimals, will be the answer in dollars, and the decimal of a dollar. If pounds only be given, annex ciphers in the place of the decimal, and proceed as above directed.

* Virginia and Kentucky currency is the same as New-England; that of North-Carolina and Ohio, the same as New-York.

Examples.

1. In £22 how many dollars, cents and mills?

.4 | .3)22.0000

Ans $\left\{ \begin{array}{l} \$73.333\text{* if N.E. cur.} \\ \$55.000 \text{ if N.Y. cur.} \end{array} \right.$

2. Reduce £16 7s. 8½d. in each currency to Federal Money.

.4|.3)16.385

$\left. \begin{array}{l} \text{N.E. } \$54.616 \\ \text{N.Y. } \$40.962 \end{array} \right\}$ Ans.

Here 3, half the greatest even number of shillings, is the first decimal figure. 8½d.=34qrs. This is increased by 1, because over 12, making it 35, and 3 is increased by 5, because the shillings were odd, making it 85.

3. Reduce £91 in each currency to Federal Money.

$\left. \begin{array}{l} \text{N.E. } \$303.333 \\ \text{N.Y. } \$227.50 \end{array} \right\}$ Ans.

4. Reduce £125 N.E. to Federal Money. Ans. $416.366.

5. Reduce £33 13s. N.Y. to Federal Money.

Ans. $84.125.

6. Reduce £25 15s. N.E. to Federal Money.

Ans. $85.833.

7. In £227 17s. 5¼d. N.E. how many dollars, cents, and mills? Ans. $759 57cts. 8m.

8. Reduce £49 1s. in each currency to Federal Money.

$\left. \begin{array}{l} \text{£49 1s. N.E.=\$163.50} \\ \text{£49 1s. N.Y.=\$122.625} \end{array} \right\}$ Ans.

9. Reduce £8 7s. 2¼d. in each currency to Federal Money.

$\left. \begin{array}{l} \text{£8 7s. 4½d.N.E.=\$27.87} \\ \text{£8 7s. 4½d. N.Y.=\$20.902} \end{array} \right\}$ Ans.

10. Reduce £111 11s. 11¼d. N.Y. to Federal Money.

Ans. $278 99cts. 2m.

11. Reduce £86 6s. 5¼d. in currency to Federal Money.

$\left. \begin{array}{l} \text{N.E. } \$287.74 \\ \text{N.Y. } \$215.805 \end{array} \right\}$ Ans.

12. In £365 N.Y. how many dollars, &c.? Ans. 912.50.

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* The reason of this rule will be obvious by considering that in N.E. currency, 6s. or $\frac{6}{20}$ of a pound, are equal to $1, and $\frac{6}{20}=\frac{3}{10}$, or .3 of a pound; and that in N.Y. currency, 8s. or $\frac{8}{20}$ of a pound=1 dollar, and $\frac{8}{20}=\frac{4}{10}$, or .4 of a pound. Hence, in the former case, there are evidently as many dollars in the pounds, as there are $\frac{3}{10}$, and in the latter, as many as there are $\frac{4}{10}$. Therefore, dividing the pounds by .3 in one case, and .4 in the other, and pointing according to the rule for the division of decimals, the quotient is evidently dollars and the decimal of a dollar.—If, after reducing the shillings, &c. to the decimal of a pound by inspection, the separatrix be removed one place to the right hand, the sum will be 2 shilling pieces and the decimal of a 2s. piece, and this divided by 3 and 4, as whole numbers will evidently give dollars and decimals of dollars in the two cases, because in the former, 3 two shillings, and in the latter, 4, are equal to a dollar.

Case II.

To reduce Federal Money to New-England, &c. to New-York, &c. currencies.

Rule.*—Multiply the given sum by .3 for New England currency, and by .4 for New-York currency, and the product, pointed according to the rule for the multiplication of decimals, will be pounds and the decimals of a pound. Then find the value of the decimal by inspection. (See Reduction of Decimals, Case V.)

Examples.

1. In $25.964, how many pounds, shillings, pence, and farthings?

25.964	25.964
.3	.4
£7.7892	£10.3856

Ans. { £7 15s. 9¼d. N.E. / £10 7s. 8½d. N.Y. }

2. In $912.50 how many pounds, shillings, pence and farthings?

Ans. { £273 15s. N.E. / £365 0s. N.Y. }

3. In 49 cents how many shillings, pence and farthings?

Ans. { 2s. 11¼d. N.E. / 3s. 11d. N.Y. }

4. In $6.753 how many pounds, shillings, pence, and farthings?

Ans. { £2 0s. 6d. N.E. / £2 14s. 0¼d. N.Y. }

5. In $1.612 how many shillings, pence and farthings?

Ans. { 9s. 8d. N.E. / 12s. 10¾d. N.Y. }

5. In $1111.111 how many pounds, shillings, pence and farthings?

Ans. { £333 6s. 8d. N.E. / £444 8s. 10½d. N.Y. }

Case III.

To reduce Pennsylvania, New-Jersey, Delaware and Maryland currency to Federal Money.

Rule.†—Reduce the given sum to pence, and if there are farthings, for 1 qr. place 2 at the right hand of the pence; for 2 qrs. write 5, and for 3, write 7; but if there are no farthings, annex a cipher to the pence. Divide this sum by 9, and add the quotient to the dividend. From the sum point off three figures for cents and mills; those on the left hand will be dollars.

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* As this rule is the converse of the preceding, the truth of it must be sufficiently obvious from what has already been said.

† A dollar in Pennsylvania, &c. currency, is 7s. 6d.=90d. which, increased by one ninth of itself, is 100= to the number of cents in a dollar. Hence the reason of the rule is obvious.

Examples.

1. In £25 6s 5d 2qrs how many dollars?

£ s d qrs
25 6 5 2
20
———
506
12
———
9)60775
6752
———
Ans. $67.527

2. In £86 6s 5½d how many dollars, cents and mills?
Ans. $230.191.

3. Reduce £736 to Federal money. Ans. $1962.666.

4. Reduce £42 12s 8d to Federal money.
Ans. $113.688.

Case IV.

To reduce Federal Money to Pennsylvania, &c. currency.

Rule.

If there be no mills in the given sum, reduce it to mills by annexing ciphers; from the sum subtract one tenth of itself, and the remainder, excepting the right hand figure, will be pence, which must be reduced to pounds. If the figure cut off from the right hand be 2, call it 1qr. if 5, 2qrs. and if 7, 3qrs.

Examples.

1. In $67.527 how many pounds, shillings, pence and farthings?

67.527
6752
———
12)6077.5
———
2|0)50|65½d.
———
£25 6s

Ans. £25 6s. 5½d.

To get one tenth, you divide by 10, and a sum is divided by 10, by cutting off the right hand figure; hence we have only to drop the right hand figure and subtract the others from the given sum.

2. In $230.191 how many pounds, shillings, pence and farthings?
Ans. £86 6s. 5¼d.

3. In $200 how many pounds?
Ans. £75.

4. In $14.25 how many pounds, shillings, pence and farthings?
Ans. £5 6s. 10½d.

Case V.

To reduce Canada currency to Federal Money.

Rule.*

If there be shillings, pence and farthings in the given sum, reduce

* Here the shillings, pence, &c. are reduced to the decimal of a pound, and the whole multiplied by 4, because £1=4 dollars; a dollar in that currency being equal to 5 shillings. The currency of Nova Scotia is the same as Canada.

them to the decimal of a pound by inspection, (See Case III. Reduction of Decimals,) and write the decimal at the right hand of the pounds; multiply the sum by 4, and the product will be dollars and decimal parts.

Examples.

1. In £6 4s 6d 2qrs how many dollars?

   6.227
   4

   $24.908 Ans.

2. In £125 9s 7d 3qrs how many dollars?

   Ans. $501.928.

Case VI.

To reduce Federal Money to Canada currency.

Rule.

Divide the given sum by 4, and the quotient will be pounds and the decimal of a pound. The value of the decimal must be found by inspection. (See Case V. Reduction of Decimals.)

Examples.

1. Reduce $24.908 to pounds, shillings, pence and farthings.

   4)24.908

   6.227

   Ans. £6 4s. 6½d.

2. In $501.928 how many pounds, shillings, pence and farthings?

   Ans. £125 9s. 7¾d.

Additional Rules in Exchange.

VII.	To change	N. E. currency to N. Y. currency; add one third.
VIII.	"	N. Y. to N. E. currency; subtract one fourth.
IX.	"	N. E. to Penn. currency; add one fourth.
X.	"	Pen. to N. E. currency; subtract one fifth.
XI.	"	N. Y. to Penn. currency; subtract one sixteenth.
XII.	"	Penn. to N. Y. currency; add one fifteenth.
XIII.	"	N. E. to Canada currency; subtract one sixth.
XIV.	"	Canada to N. E. currency; add one fifth.

Miscellaneous Examples.

1. In £1 1s 10½d N. E. currency, how many dollars?
Ans. $3.646.

2. In £1 1s 10½d N. Y. currency, how many dollars?
Ans. $2.735.

3. In £1 1s 10½d Penn. currency, how many dollars?
Ans. $2.916.

4. In £1 1s 10½d Canada currency, how many dollars?
Ans. $4.376.

5. In $255.406 how many pounds, shillings, pence and farthings?

Ans. { £76 12s 5d N. E. cur.
£102 3s 3d N. Y. cur.
£95 15s 6¼d Penn. cur.
£63 17s 0¼d Canada cur. }

6. Change £240 15s N. E. currency to the several other currencies.

Ans. { £321 0s 0d N. Y. cur.
£300 18s 9d Penn. cur.
£200 12s 6d Canada cur.
$802.50 Federal Money. }

QUESTIONS.

1. How do you reduce N. E. and N. Y. currencies to Federal Money?
2. In what other States is the currency the same as in New-England?
3. In what the same as New-York?
4. How do you change Federal Money to N. E. and N. Y. currencies?
5. How do you change Pennsylvania currency to Federal Money?
6. In what other States is the currency the same as Pennsylvania?
7. How do you change Federal Money to Pennsylvania currency?
8. How is Canada currency reduced to Federal Money?
9. How is Federal Money reduced to Canada currency?
10. How is N. E. currency changed to N. Y. currency?—N. Y. to N. E.?—N. E. to Penn.?—Penn. to N. E.?—N. Y. to Penn.?—Penn. to N. Y.?—N. E. to Canada?—Canada to N. E.?

SECTION IV.

Proportion.

1. SINGLE RULE OF THREE.

The Single Rule of Three is known by having *three* numbers given to find a *fourth*, which shall bear the same proportion to the *second* that the *third* has to the *first*.

It is sometimes called *Simple Proportion*, and, on account of its importance, the *Golden Rule*.

Rule.*

1. Write down that number which is of the same kind with the answer, or number sought, for the second term.

2. Consider whether the answer ought to be greater or less than this number, and if *greater*, place the greater of the other two given numbers for the third term, and the less for the first term; but if *less*, write the less of the other two given numbers for the third term, and the greater for the first.

3. Multiply the second and third terms together, and divide the product by the first, the quotient will be the answer.

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* Proportion is of two kinds; one arises from considering the differences of numbers, and is called *Arithmetical Proportion;* the other from considering their quotients, and is called *Geometrical Proportion*. The latter is that with which we are at present concerned. Four numbers are said to be in geometrical proportion when the *first* has the same proportion to the *second* which the *third* has to the *fourth*; that is, when the quotient of the second, divided by the first, is the same as the quotient of the fourth, divided by the third, and the reverse. Thus 2 : 4 : : 6 : 12 are in geometrical proportion, because 2 is to 4, as 6 to 12, that is, 4 divided by 2 gives the same quotient as 12 divided by 6: viz. 2; and if 2 be divided by 4, and 6 by 12, the quotient is in each case .5. In the same way it will appear that the fourth term has the same proportion to the second as the third has to the first, and the reverse. It also appears, that the product of the first and fourth terms, or extremes, ($2\times12=24$,) is equal to the product of the second and third, or means ($4\times6=24$.) This holds true in all cases where the numbers are proportional, and upon this fact, all the operations in the rule of three are founded.

In order to compare numbers together, it is necessary to consider them abstractly, or as applied to things of the same kind. We cannot compare 2 men and 4 days, but we may compare 2 and 4, or 2 men and 4 men, or 2 days and 4 days. In the Rule of Three, we have three terms given to find a fourth, which shall have the same relation to one of the given terms which exists between the other two. Two of the given terms will therefore apply to things of the same kind, so as to be compared; and the other known term and the unknown term will also apply to similar objects, so that a like comparison may be instituted between them. Example.—If 2lb. of sugar cost 14cts. what will 12lb. cost? Here 2 and 12 apply to pounds of sugar, they may therefore be compared; and

Proof.

Invert the order of the question, and proceed as before.

N. B. Before stating the question, the first and third terms must be reduced to the same denomination, if they are not already so, and the middle term to the lowest denomination mentioned in it. The answer will be in the same denomination as the second term, and may be brought to a higher by reduction if necessary.

Examples.

1. If 15 bushels of corn cost $7.50, what will 25 bushels cost?

```
bu.  $cts.  bu.
15 : 7.50 : : 25
       25
    ------
     3750
    1500
    ------  $ cts.
15)187.50(12.50 Ans.
```

Here it will be seen at once that the answer is to be in money, and therefore, $7.50 must be the second term. It is also evident the 25 bushels will cost more than 15, and therefore that the answer will be more than $7.50, and consequently, 25 must be the third term, and 15 the first.

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the required number must be cents in order to compare with 14. Now it is evident that the cost of 12lb. will be as many times 14cts. as 12 is times 2, and therefore, the number expressing the value of 12lb. will bear the same proportion to 14 that 12 does to 2; and 2, 14 and 12 will be the three first terms of a geometrical proportion; that is, 14 and 12 will be the two means, and 2 the first extreme. Now since the product of the two means is equal to the product of the extremes, it is plain that if the product of the means be divided by one extreme, the quotient will be the other extreme; thus $14\times12=168$, product of means, and $168\div2=84$, the other extreme, which is precisely the rule. If from the nature of the question, the answer is required to be greater than the given number of the same kind, that number must evidently be multiplied by the greater of the other two given numbers, and the product divided by the less, and the reverse when the answer is required to be less. Hence the direction for stating is obvious.

Besides the method given above for performing the operation in the Rule of Three, there are the four following.

1. Divide the second term by the first, multiply the quotient by the third, and the product will be the answer.
2. Divide the third term by the first, multiply the quotient by the second, and the product will be the answer.
3. Divide the first term by the second, divide the third term by the quotient, and the last quotient will be the answer.
4. Divide the first term by the third, divide the second by the quotient, and the last quotient will be the answer.

The Single Rule of Three is usually divided into *Direct* and *Inverse*, both of which are included in the general rule given above. But for the sake of such as may wish to acquaint themselves with proportion, considered under these two divisions, they are here subjoined.

I. *Single Rule of Three Direct* teaches by having three numbers given to find a fourth, which shall have the same proportion to the third as the second has to the first.

RULE.—1. State the question by making that number which asks the ques-

2. If $7.50 buy 15 bushels of corn, what will $12.50 buy?

$cts. bu. $ cts.
7.50 : 15 : : 12.50
15
———
6250
1250
——— bu.
7.50)187.50(25 Ans.

This is the reverse of the first example, and therefore proves it.

3. If a family of 12 persons spend 5 bushels of wheat in 4 weeks, how much will last them a year, allowing 52 weeks to a year?

w. bu. w.
4 : 5 : : 52 Ans. 65 bush.

4. If 8lb. 4oz. of tobacco cost 5s. 6d. what will 24lb. 12oz. cost?

lb.oz. s.d. lb.oz.
8 4 5 6 24 12
16 12 16
——— ——— ———
132oz. 66d. 156
24
———
396oz.

Here the several terms must be reduced to the lowest denominations mentioned, before stating the question.

oz. d. oz.
132 : 66 : : 396
66
———
2376
2376
——— d.
132)26136(198=16s. 6d. Ans.

---

tion, the third term, that which is of the same kind, the first term, and that which is of the same kind as the answer, the second term. 2. Multiply the second and third terms together, and divide the product by the first, the quotient will be the answer.

EXAMPLE.—If 8lb. of sugar cost $1.00, what will 40lb. of sugar cost?

8 : 1.00 : : 40
40
———
8 (40.00
———
$5 00 Ans.

Here 40 asks the question, it is therefore the third term; 8 being of the same kind, is the first; and 1.00 being of the same kind as the answer, is the second.

II. The *Single Rule of Three Inverse* teaches by having three numbers given to find a fourth, which shall bear the same proportion to the second that the first has to the third.

RULE.—State the question as in the rule of three direct. Multiply the first and second terms together, and divide the product by the third, the quotient will be the answer.

EXAMPLE.—How many yards of sarcenet 3qs. wide, will line 9 yards of cloth 8qrs. wide?

8 : 9 : : 3
8
———
3) 72
———
24 Ans.

Here 3 asks the question, 8 is of the same kind, and 9 the same as the answer sought. Therefore the product of 8 and 9 divided by 3, is the answer.

Having stated the question, to know whether it belongs to inverse or direct proportion.—1. If the third term be greater than the first, and the fourth term is required to be greater than the second, or if the third term be less than the first, and the fourth term is required to be less than the second, the question belongs to the rule of three direct. 2. If the third term be greater than the first, and the fourth term is required to be less than the second, or if the third term be less than the first, and the fourth term is required to be greater than the second, the question belongs to the rule of three inverse.

5. If 8 acres produce 176 bushels of wheat, what will 34 acres produce?

Ans. 748 bushels.

6. If 9lb. of sugar cost 6s. what will 25lb cost? Ans. 16s. 8d.

When there is a remainder after dividing the product of the second and third terms by the first, reduce it to the next lower denomination, and divide as before.

7. A borrowed of B $250 for 7 months; afterwards B borrowed of A $300; how long must he keep it to balance the former favor? Ans. 5ms 25ds.

8. If 100 men can do a piece of work in 12 days, how many men can do the same in 3 days?

Ans. 400 men.

9. A goldsmith sold a tankard weighing 39oz 15pwt for £10 12s; what was it per ounce?

oz. pwt. £
39 15 : 10 12 : : 1 Ans. 5s. 4d.

11. If the interest of $100 for one year be $6, what will be the interest of $336 for the same time? $ $ $
100 : 6 : : 336 Ans. $20.16.

11. If $100 gain $6 in one year, in what time will a sum of money double at that rate, simple interest?

$ yr. $
6 : 1 : : 100 Ans. 16⅔ yrs.

12. If $100 gain $6 in 12 months, in how many months will a sum of money double at that rate, simple interest?

$ mo. $
6 : 12 : : 100 Ans. 200mo.

13. If $100 gain $6 in 365 days, in how many days will a sum of money double at that rate, simple interest?

Ans. 6083⅓ days.

14. A owes B £296 17s. but becoming a bankrupt, can pay only 7s 6d on the pound; how much will B receive?

Ans. £111 6s 4d 2qrs.

15. If one dozen of eggs cost 10½ cents, what will 250 eggs cost? Ans. $2.187.

16. If a penny loaf weigh 9oz when wheat is 6s 3d per bushel, what ought it to weigh when wheat is 8s 2½d per bushel?

Ans. 6oz 13drs.

17. How many yards of flannel 5qrs wide, will line 20 yards of cloth 3qrs wide?

Ans. 12 yards.

18. If a staff 4ft 6in in length, cast a shadow 6 feet, what is the height of a tree whose shadow measures 108 feet?

Ans. 81 feet.

19. If the earth revolve on its axis 366 times in 365 days, in what time does it perform one revolution? Ans. 23h 56m
rev. ds. rev. 4s nearly.*
366 : 365 : : 1

20. If a person at the equator be carried by the diurnal motion of the earth, 25000 miles in 24 hours, how far is he carried in a minute? Ans. $17\frac{13}{36}$ miles.

21. There is a cistern which has three cocks; the first will empty it in ¼ of an hour, the second in ¾ of an hour, and the third in 1½ hour; in what time will they all empty it running together?

h cis h cis Ans. 10m.
.25 : 1 : : 1 : 4
.75 : 1 : : 1 : 1.333+
1.50 : 1 : : 1 : 0.666+

6 cisterns.
cis m cis m
6 : 60 : : 1 : 10 Ans.

* This is called a sideral day.

22. Bought 4 bales of cloth, each containing 6 pieces, and each piece containing 27 yards, at £16 4s per piece; what is the value of the whole and the price per yard?
Ans. £388 16s and 12s per yd.

23. If a hogshead of rum cost $75.60, how much water must be added to it to reduce the price to 1 dollar per gallon?
Ans. 12⅗ gal.

24. If a board be 9 inches wide, how much in length will make a square foot? Ans. 16in.
9 : 144 : : 1

25. How many yards of paper 3 quarters of a yard wide, will paper a room that is 24 yards round, and 4 yards high?
Ans. 128 yards.

26. The salary of the President of the United States is $25,000 a year; what is that per day? Ans. $68·493.

27. A garrison of 500 men has provisions for 6 months; how many must depart that there may be provisions for those who remain, 8 months? Ans. 125.

28. If a man spend 75 cents per day, what does he spend per annum? Ans. $273,75.

29. If a field will feed 6 cows 91 days, how long will it feed 21 cows? Ans. 26 days.

30. A lends B $66 for one year; how much ought B to lend A for 7 months, to balance the favor? Ans. $113.142.

31. At $1,25 per week, how many weeks' board can I have for $100? Ans. 80 weeks.

32. What will 39 weeks' board come to at $1,17 per week?
Ans. $45,63.

33. If my watch and seal be worth 48 dollars, and my watch be worth 5 times as much as my seal, what is the value of the watch? Ans. $40.
6 : 48 : : 5

34. A cistern containing 230 gallons, has two pipes; by one it receives 50 gallons per hour, and by the other discharges 35 gals. per hour; in what time will it be filled? Ans. 15h. 20m.

35. If 40 rods in length and 4 in breadth make one acre, how many rods in breadth, that is 16 rods long, will make one acre?
Ans. 10 rods.

36. The earth is 360° in circumference, and revolves on its axis in 24 hours; how far does a place move in one minute in lat. 44°, a degree in that lat. being about 50 miles?
Ans. 12½ miles.

h m deg m m
$24 \times 60 : 360 \times 50 : : 1$

37. If the earth perform its diurnal revolution in 24 hours, in what time does a place on its surface move through one degree?
360° : 24 : : 1° Ans. 4 minutes.

38. There is a cistern which has a pipe that will empty it in 6 hours; how many such pipes will be required to empty it in 20 minutes? Ans. 18 pipes.

39. What is the value of $642 against an estate which can pay only 69 cents on the dollar?
Ans. $442,98.

40. How many men must be employed to finish in 9 days, what 15 would do in 30 days?
Ans. 50 men.

RULE OF THREE IN VULGAR FRACTIONS.

Prepare the fractions by reduction, if necessary, and state the question by the general rule; invert the first term, and then multiply all the numerators together for a new numerator, and all the denominators together for a new denominator; the new numerator, written over the new denominator, will be the answer required.

Examples.

1. If $\frac{5}{7}$ oz. cost $\frac{7}{9}$£, what will 1oz. cost?

oz. £ oz.

$\frac{5}{7} : \frac{7}{9} :: 1$ Then,

$\frac{7}{5} \times \frac{7}{9} \times 1 = \frac{49}{45}$£ = £1 1s. 9⅓d. An.

2. How much shalloon that is ¾yd. wide, will line 13¼ yards of cloth that is 2½ yard wide?

$13\frac{1}{4} = \frac{53}{4}$ and $2\frac{1}{2} = \frac{5}{2}$ $\frac{3}{4} : \frac{53}{4} :: \frac{5}{2}$

$\frac{4}{3} \times \frac{53}{4} \times \frac{5}{2} = \frac{1060}{24}$ = 44 yds. 6 in. Ans.

3. If $\frac{5}{8}$ gallon cost $\frac{5}{8}$ £ what will $\frac{5}{9}$ tun cost?

$\frac{5}{8}$ of $\frac{1}{63}$ of $\frac{1}{2}$ of $\frac{1}{2} = \frac{5}{2016}$ tun.

$\frac{5}{2016} : \frac{5}{8} :: \frac{5}{9}$ Ans. £140.

4. If my horse and chaise be worth $175, and the value of my horse be $\frac{3}{2}$ that of my chaise, what is the value of each?

$1 : \frac{175}{1} :: \frac{3}{5}$: $105 horse.

$1 : \frac{175}{1} :: \frac{2}{5}$: $70 chaise.

Ans.

5. A lends B $48 for ¾ of a year; how much must B lend A $\frac{5}{12}$ of a year to balance the favor? Ans. $86.40.

6. A person owning $\frac{3}{5}$ of a farm sells ¾ of his share for £171; what is the whole farm worth? Ans. £380.

*ASSESSMENT OF TAXES.**

1. Supposing the Legislature should grant a tax of $35000 to be assessed on the inventory of all the rateable property in the State, which amounts to $3000000 what part of it must a town pay, the inventory of which is $24600?

dol.inv. dol.tax. dol.inv. dol.

3000000 : 35000 :: 24600 : 287 Ans.

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* In assessing taxes, it is generally best, first to find what each dollar pays, and the product of each man's inventory, multiplied by this sum, will be the amount of his tax. In this case, the sum on the dollar, which is to be employed as a multiplier, must be expressed as a proper decimal of a dollar, and the product must be pointed according to the rule for the multiplication of decimals; thus 2 cents must be written .02, 3 cents, .03, 4 cents, .04, &c. It is sometimes the practice to make a table by multiplying the value on the dollar by 1, 2, 3, 4, &c. as follows:

TABLE.

1	is	3	10	is	30	100	is	3 00
2	—	6	20	—	60	200	—	6.00
3	—	9	30	—	90	300	—	9.00
4	—	12	40	—	1.20	400	—	12.00
5	—	15	50	—	1.50	500	—	15.00
6	—	18	60	—	1.80	600	—	18.00
7	—	21	70	—	2 10	700	—	21.00
8	—	24	80	—	2.40	800	—	24.00
9	—	27	90	—	2.70	900	—	27.00
10	—	30	100	—	3.00	1000	—	30.00

2. A certain school, consisting of 60 scholars, is supported on the polls of the scholars, and the quarterly expense of the whole school is $75; what is that on the scholar, and what does A pay per quarter, who has 3 scholars?

Ans. $1.25 on the scholar, and A pays $3.75 per quarter.

3. If a town, the inventory of which is $24600, pay $287, what will A's tax be, the inventory of whose estate is $525.75?

dol.inv. Tax. dol.inv.
24600.00 : 287 :: 525.75 : $6.133 Ans.

4. The inventory of a certain school district is $4325, and the sum to be raised on this inventory for the support of schools, is $86.50; what is that on the dollar, and what is C's tax, whose property inventories at $76.44?

$4325 : 86.50 :: 1 : .02cts. Ans. and 76.44 × .02 = $1.528 C's tax.

5. If a town, the inventory of which is $16436, pay a tax of $493.08, what is that on the dollar?

$16436 : $493.08 :: 1 : .03cts. Ans.

QUESTIONS.

1. How is the Single Rule of Three known?
2. By what other names is it sometimes called?
3. Which of the given numbers is to be written down for the second term?
4. How is it to be determined which of the others is to possess the third place?
5. How do you proceed, after stating the question, to find the answer?
6. If the first and third terms be of different denominations, what is to be done?
7. What is to be done, if the second term contain different denominations?
8. Of what denomination will the quotient be?
9. If the quotient be not of the denomination in which the answer is required, what is to be done?
10. How are two sums in Federal Money brought to the same denomination? (Ans. By annexing ciphers.)
11. What is the method of proof in this rule?
12. How is the Rule of Three performed in Vulgar Fractions?

This table is constructed on the supposition that the tax amounts to three cents on the dollar, as in example 3d. USE.—What is B's tax, whose rateable property is $276? By the table it appears that $200 pay $6, that $70 pay $2.10, and that $6 pay 18 cents.

Thus $200	is $6 00	
70	— 2.10	
6	— 0.18	
276	$8.28	B's tax.

Proceed in the same way to find each individual's tax, then add all the taxes together, and if their amount agree with the whole sum proposed to be raised, the work is right. It is sometimes best to assess the tax a trifle larger than the amount to be raised, to compensate for the loss of the fractions.

2. DOUBLE RULE OF THREE.

The Double Rule of Three, sometimes called *Compound Proportion,* teaches to resolve such questions as require two or more statements in the Single Rule of Three.

In the Double Rule of Three, there are usually five numbers given to find a sixth, but there may be more, as seven or nine.

Rule.

1. Make the number, which is of the same kind as the required answer, the second term.

2. Take any two of the remaining numbers that are of the same kind, and place one for a first term, the other for a third term, according to the directions given in the Single Rule of Three; then take any other two of the same kind, and place them in the same way, and so on, till all are used.

3. Multiply the product of the third terms by the second term, and divide the result by the product of the first terms, and the quotient will be the answer required.

Examples.

1. A wall which is to be built to the height of 27 feet, was raised to the height of 9 feet by 12 men in 6 days; how many men must be employed to finish the wall in 4 days?

27—9=18

9 : 12 : : 18
4 :   : 6
——   ——
36   108
     12
     ——
36)1296(36 Ans.
    108
    ——
    216
    216
    ——

By taking 9 from 27, we find the wall is to be raised 18 feet higher, and it is required to know how many men will finish it in 4 days; consequently 12 must be made the second term. Now if we take two numbers of the same kind, viz. 9 and 18, it is plain that the 18 will require more men than the 9, supposing the time to be the same; then 9 must stand in the first place, and 18 in the third. Again; if we take the other two of the same kind, viz. the 6 and 4, and suppose the work the same, it is evident that if the same work is required to be done in less time, the number of men must be increased; i. e. if the work done by 12 men in 6 days is to be done in 4 days more than 12 men must be employed; hence 6 must stand in the third place, and 4 in the first.

2. If a man travel 112 leagues in 29 days, when the days are 7 hours long, how far will he travel in 17 days, when they are 10 hours long?

29 : 112 : : 17
 7 :   : : 10

The days, their length being considered equal, would require the answer to be less than 112, because in this case he could not travel so far in 17 as in 29 days; and the hours, without regarding the days, would require the answer to be greater than 112, for he could certainly travel further in 10 than in 7 hours, and these two compounded as in the statement, give the distance required.

3. If 120 bushels of oats will serve 14 horses 56 days, how many days will 94 bushels serve 6 horses?

Ans. $102\frac{26}{45}$ days.

4. If $100 gain $6 in 12 mo. what will be the interest of $350 for 2 years and 7 months?

	$	mo.		$
2y. 7mo.=31mo.	100	: 6	::	350
	12	:	::	31

Ans. $54.25.

5. If a sum of money at 6 per cent, simple interest, double in 200 months, what will be the interest of $300 for 8 months?

$	mo.		$
100	: 100	::	300*
200	:	::	8

Ans. $12.

6. If the transportation of 20 cwt. 37 miles cost $16, what will the transportation of 12 cwt. 50 miles cost? Ans. $12.972.

7. If the interest of $45 for 6 months be $1.80, what is the rate per annum?

Ans. 8 per cent.

8. If 8 men spend $48 in 24 weeks, how much will 40 men spend in 48 weeks at the same rate? Ans. $480.

9. If the freight of 5 tierces of salt, each weighing $5\frac{1}{2}$ cwt. 80 miles, cost $28, what will be the freight of 75 sacks of salt, each weighing $2\frac{1}{4}$ cwt. 150 miles?

Ans. 322.159\frac{1}{11}$

10. A man lent $350 to receive interest, and when it had continued 9 months, he received, principal and interest together, $360.50; at what rate per cent did he lend his money?

Ans. 4 per cent.

11. With how many pounds sterling could I gain £5 per annum, if with £450 I gain in 16 months, £30? Ans. £100.

12. If 1 lb. of thread make 3 yards of linen 5 qrs. broad, how many pounds of thread will be wanted to make a piece of linen 45 yards long and 1 yard broad?

Ans. 12 lb.

13. If £8 is gained in 12 months with £100, with how much money can I gain £8 12s. in 5 months? Ans. £258.

14. If 200 lb. of merchandize are carried 40 miles for 3 shillings, how many pounds may be carried 60 miles for £22 14s. 6d?

Ans. 20200 lb.

15. If for 3 shillings 200 lb. of goods are carried 40 miles, how many miles may 20200 lb. be carried for £22 14s. 6d.?

Ans. 60 miles.

16. If 200 lb. of goods are carried 40 miles for 3 shillings, how much must be paid for carrying 20200 lb. 60 miles?

Ans. £22 14s. 6d.

---

* On this statement is founded one of the practical rules for computing interest, as will be hereafter shown.

3. CONJOINED PROPORTION.

Conjoined Proportion is when the coins, weights, or measures of several countries are compared in the same sum.

Case I.

To find how many of the last kind of coin, weight, or measure, mentioned in the question, are equal to a given number of the first.

Rule.*—Make the given number the third term. Of the other numbers, multiply all the antecedents together for the first term, and all the consequents together for the second; then state the question, and proceed as in the Single Rule of Three.

Examples.

1. If 10lb. at Boston make 9lb. at Amsterdam, and 90lb. at Amsterdam 112lb. at Thoulouse; how many pounds at Thoulouse are equal to 50lb. at Boston?

Ant.	Con.
10	9
90	112

lb.
900 : 1008 :: 50 given number.
50

900)50400(56 Ans.
4500

5400
5400

2. If 20 braces at Leghorn equal 10 vares at Lisbon, and 40 vares at Lisbon 80 braces at Lucca, how many braces at Lucca are equal to 100 braces at Leghorn? Ans. 100 braces.

3. If 40lb. at New-York make 36 at Amsterdam, and 90lb. at Amsterdam make 116 at Dantzick; how many pounds at Dantzick are equal to 244 at New-York? Ans. $233\frac{12}{25}$.

4. What will 1 lb. of pepper cost, if 3 lb. of cloves cost as much as 6 lb. of pepper, and $2\frac{1}{2}$ lb. of cinnamon cost as much as 4 lb. of cloves, and 3 lb. of cinnamon cost 8 shillings? Ans. 10d.

5. If 10 lb. at London be equal to 9 lb. at Amsterdam, 45 lb. at Amsterdam to 49 lb. at Bruges, and 98 lb. at Bruges to 116 lb. at Dantzick; how many pounds at Dantzick are equal to 112 lb. at London? Ans. $129\frac{46}{50}$ lb.

6. If 3 yards of cloth cost as much as $3\frac{3}{4}$ yards of ratteen, and $4\frac{3}{8}$ yards of ratteen are worth 5 yards of druggit; how many yards of druggit are worth 27 yards of cloth? Ans. $37\frac{4}{7}$ yds.

* This and the following rule may often be abridged by cancelling where the same number is found among the antecedents and consequents. The proof is by several statements in the Single Rule of Three.

Case II.

To find how many of the first kind of coin, weight, or measure, mentioned in the question, are equal to a given number of the last.

Rule.—Proceed as in the first case, only make the product of the consequents the first term, and that of the antecedents the second.

Examples.

1. If 100lb. in America make 95lb. Flemish, and 19lb. Flemish 25lb. at Bolognia, how many pounds in America are equal to 50lb. at Bolognia?

Ant.	Con.	lb.		lb.		lb.
100	95	2375	:	1900	::	50
19	25			50		
1900	475			lb.		
	190	2375)95000(40 Ans.				
	2375			9500		
				0		

2. If 6 braces at Leghorn make 3 ells English, and 5 ells English 9 braces at Venice, how many braces at Leghorn will make 45 braces at Venice?

Ans. 50 braces.

3. If 20lb. at Boston make 23 at Antwerp, and 155lb. at Antwerp make 180 at Leghorn; how many pounds at Boston are equal to 144 at Leghorn?

Ans. $107\frac{1}{2}\frac{2}{3}$lb.

QUESTIONS.

1. By what other name is the Double Rule of Three sometimes called?
2. What does this rule teach?
3. How many numbers are there usually given?
4. In stating the question, how are the three conditional terms placed?
5. What is to be done with the other two terms?
6. If the blank fall under the third term, how do you proceed?
7. How, if it fall under the first or second term?
8. What is Conjoined Proportion?
9. How do you proceed, when it is required to find how many of the last kind of coin, weight, or measure, are equal to a given number of the first?
10. How do you proceed when it is required to find how many of the first kind are equal to a given number of the last?

SECTION V.

Interest.

Interest is a premium allowed for the use of money. It is computed at so many dollars a year for the use of each hundred dollars, called so much *per cent. per annum.*

The *principal* is the sum which is upon interest.

The *rate** is the per cent. per annum agreed on.

The *amount* is the principal and interest added together.

Interest is of two kinds, *Simple* and *Compound.*

1. *SIMPLE INTEREST.*

Simple Interest is that which is allowed for the principal only.

Case I.

To find the interest on any sum in Federal or English money.

Rule.†

1. Multiply the principal by the rate, and divide the product by 100, the quotient will be the interest for one year.
2. Multiply the interest thus found, by the number of years, and the product will be the interest for that time.
3. If there be *months* and *days,* for the former take proportional parts of the interest for one year, and for the latter, proportional parts of the interest for one month, allowing 30 days to a month.

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* The rate is generally established by law. Six per cent. is the legal interest in the several New-England States, and this is to be understood in this work, where the rate is not mentioned. In New-York, legal interest is 7 per cent.

† This rule is barely an application of the Single Rule of Three, or saying as 100, or £100, is to the rate, so is the principal to the interest for one year. Example.—What is the interest of $250 for one year, at 6 per cent? As $100 : 6 : : 250 : Ans. $15. The reason for the remaining part of the rule must be obvious. When the months are not an aliquot part of a year, divide them into two such parts as shall be aliquot parts of a year, find the interest of those two, and add them together. The same may be done when the days are not an aliquot part of a month.

Examples.

1. What is the interest of $48.643 for 2 years, at 6 per cent per annum?

```
        Principal 48.643
            Rate.      6
                 -------
             1|00)2|91.858
   1 year's int. 2.91858
                       2
                 -------
2 yr's int. $5.83716 Ans.
```

To divide by 100, we have only to remove the separatrix two figures from its natural place, towards the left hand. Here the answer is found to be 5 dollars, 83 cents, 7 mills, and 16 hundredths of a mill. All below mills is usually rejected in practical operations.

2. What is the interest of $225.755 for 3 years, 8 months, and 10 days, at 6 per cent?

```
                  225.755
                        6
                 --------
6 mo.=1/2)13.54530 interest for 1 year.
                        3
                 --------
          40.63590 interest for 3 years.
2 mo.=1/3)6.77265 interest for 6 months.
 10d.=1/6)2.25755 interest for 2 months.
           .37625 interest for 10 days.
        ---------
        $50.04235 Ans.
```

3. What is the interest of £86 10s. 4d. for 1 year and 6 months, at 6 per cent?

```
       £  s. d.
      86 10  4
             6
     ---------
   £5.19  2  0
      20
     ----
   s. 3.82
      12
     ----
   d. 9.84
       4
     ----
 qrs. 3.36
```

The scholar will observe that pointing off the two right hand figures for decimals, and then reducing them to the next inferior denomination, and pointing off as before, is in effect dividing by 100.

```
                 £  s.  d. qrs.
       6 mo.=1/2)5   3   9  3 interest for 1 year.
                 2  11  10  3 interest for 6 months.
               -------------
               £7  15   8  2 Ans.
```

4. What is the interest of \$175.62 for 1 year and 6 months, at 6 per cent? Ans. \$15.805.

5. What is the interest of £1 13s. 4d. for 1 year, at 9 per cent? Ans. 3s.

6. What is the interest of £25 for 6 months, at 4 per cent? Ans. 10s.

7. What is the amount of \$10.15 on interest 12 years at 6 per cent? Ans. \$17.458.

Case II.

When there are years, months, and days in the time.

Rule.*—Reduce the months and days to the decimal of a year; find the interest for 1 year by the preceding case, and multiply it by the years with the decimal annexed; the product will be the interest.

Examples.

1. What is the interest of \$6.22 for 2 years, 6 months and 10 days, at 6 per cent?

$$\frac{6}{12} = .5 \qquad \frac{10}{360} = .027$$

6m.10d. = .527+

```
  6.22
     6
 -----
 .3732
 2.527 years.
 -------
  26124
  7464
 18660
 7464
 -------
 .9430764 or,
```

94cts. 3m. Ans.

2. What is the interest of \$213.23 for 3 years and 12 days, at 10 per cent? Ans. \$64.679.

3. What is the interest of \$1600 for 1 year and 3 months, at 6 per cent? Ans. \$120.

4. What is the interest of \$121.11 for 2 years and 7 months, at 5 per cent? Ans. \$15.643.

Case III.

To cast the interest on any sum at 6 per cent.

Rule.†—1. Under the principal write half the even number of months, (with .5 at the right hand when there is an odd month) for a multiplier, by which multiply the principal and the product, after

*Months are reduced to the decimal of a year by dividing them by 12, and days to the same decimal by dividing them by 360; which is considering the month 30 days, and the year 360, and is generally practised. If greater accuracy is required, find the number of days in the given months and days, and divide them by 365, the quotient will be the true decimal of a year.

† This rule is a contraction of the Double Rule of Three. Any sum at 6 per cent. simple interest, doubles in 200 months; hence we may say, if \$100

removing the separatrix two figures from its natural place towards the left hand, will be the interest in dollars and parts of a dollar.

2. If there be days or an odd month and days in the given time, divide the days (calling the odd month 30) by 6, and place the quotient as a decimal on the right hand of half the even number of months, and proceed as before.

3. If the number of days be less than 6, supply the decimal place with a cipher, and divide the multiplicand according to the following table, and add the quotient to the product. Divide in the same manner, when there is a remainder after dividing the days by 6.

TABLE.

For 1 day, $=\frac{1}{6}$,	divide	the multiplicand		by 6.
For 2 — $=\frac{1}{3}$,	"	"	"	by 3.
For 3 — $=\frac{1}{2}$,	"	"	"	by 2.
For 4 — $=\frac{2}{3}$,	"	"	"	by 3 twice.
For 5 — $=\frac{1}{2}$ and $\frac{1}{3}$,	"	"	"	by 2 and 3.

---

in 200 months gain $100, what will a given principal gain in a given number of months? EXAMPLE.—What is the interest of $300 for 8 months, at 6 per cent?

```
            $100 : 200m. :: $100
             300 :   8
               8
            ----
            2400
             100
            ----
100×200=20000)240000(12dolls. Ans.
            20000
            -----
             40000
             40000
             -----

      mo.    $    mo.
      200 : 300 :: 8
              8
           -----
      200)2400($12 Ans. as before.
          200
          ---
          400
          400
          ---
```

Here it will be seen that the three first terms are invariable, and the two last variable quantities; and also that the first and third terms are equal, and therefore cancel each other. Hence we discover that 200 is to any given principal as the number of months in the given time is to the interest. Take the foregoing example. Here it appears that the principal is multiplied by the whole number of months, and divided by 200 for the interest. Now let us divide the third term by the first, and multiply the second term by the quotient ($8 \div 200 = .04$ and $\$300 \times .04 = 12$.) and the result is still the same. Or if we cut off the ciphers in the first term, divide the third term by 2, and multiply the second term by the quotient, cutting off the two right hand figures of the product $8 \div 2 = 4$, and $300 \times 4 = 12|00$) the result is still the same; and this last is precisely the rule, for taking half the number of months is dividing by 2, and removing the separatrix in the product, makes the result the same as if the months had been divided by 200. By this rule, half the even number of months are so many units, one month is therefore $\frac{1}{2}$, or .5, which is obtained by dividing 30 days by 6; and if any number of days less than 60 be divided by 6, the quotient may be considered a decimal of a unit, the value of which is 2 months, and may be found to any degree of exactness by annexing ciphers. But since by the rule we obtain only one decimal figure, if there is a remainder, it is necessary to take aliquot parts of the multiplicand for the odd days.

Examples.

1. What is the interest of $275.756, for 1 year, 9 months and 15 days?

3 days=½ Div. 2)275.756 10.7 ——— 1930292 2757560 137878 ——— Ans. $29.643770 or $29.64cts. 3m.	1 yr. 9mo.=21mo. and half the greatest even number is 10, and 1mo. 15ds.=45 days, which contain 6 seven times and 3 over. I therefore write .7 at the right hand of 10 in the multiplier and for the 3 days, divide the multiplicand by 2. In the product I point off two more places for decimals than there are in the multiplfcand and multiplier counted together.

2. What is the interest of 137 dollars, 84 cents, for 2 years and 6 months? Ans. 20dls. 67cts.6m.

3. What is the interest of 575 dollars for 8 months?
Ans. 23 dollars.

4. What is the interest of 13 dolls. 41cts. for 3 months and 16 days? Ans. 23cts. 6m.

5. What is the interest of 49 dollars, 25 cents for 3 years, 3 months and 3 days?
Ans. 9dolls. 62cts. 8m.

6. A note for 500 dollars on interest, was dated Sept. 22,1820, what was due, principal and interest, July 29, 1823?

	yr.	mo.	d.	Ans. $585.583.
	1823	6	29	
	1820	8	22	
Time.	2	10	7	

7. What is the amount of 212 dollars on interest for 14 months?
Ans. 226 dolls. 84cts.

8. A note for 27 dollars, 55cts. on interest was dated Feb. 14, 1823; what was there due, principal and interest, Jan. 20, 1824?
Ans. 29dolls. 9cts. 2m.

9. What is the amount of 87 dollars, 91 cents on interest 3 years and 27 days?
Ans. 104dolls. 12cts. 9m.

10. What is the interest of 607 dolls. 50cts. for 5 years?
Ans. 182dolls. 25cts.

Case IV.

To find the interest for short periods of time, at six per cent.

Rule.*—Multiply the principal by the time in days, (calling each year 360, and each month 30 days,) and divide the product by 6;

* This will be found a very convenient practical rule for casting interest for short periods of time, and it is easily remembered. The invention of the rule is similar to the preceding, dividing by 6 and removing the separatrix three figures towards the left hand, being the same as dividing by 6000 days, the number in 200 months, of 30 days each.

the quotient, after removing the separatrix three figures from its natural place towards the left hand, is the interest in dollars and parts of a dollar.

Examples.

1. What is the interest of 17 dollars, 68 cents for 11 months and 28 days?

11 mo	17.68
30	358
330 days	1444
28	8840
	5304
358 days.	6)6329.44
	$1.05440 Ans.

Here the separatrix naturally falls between 4 and 4; I therefore count off three more figures towards the left hand, and place the point between 1 and 0, and the answer is 1 dollar, 5 cts. 4 m.

2. What is the interest of 215 dolls. for 1 month and 14 days?
Ans. 1doll. 57cts. 6m.

3. What is the interest of 655 dolls. for 7 days?
Ans. 76cts. 4m.

4. What is the interest of 76 dolls. 25cts. 6m. for 1 year, 3 months and 5 days?
1yr. = 360 Ans. $5.782.
3mo. = 90
5
455 days.

When the interest is any other than 6 *per cent*; first find the interest at 6 per cent. by Case III. or IV. of which take aliquot parts and add to, or subtract from the interest at 6 *per cent.* as the case may require.

Examples.

1. What is the interest of 165 dolls. 45cts. for 1 year and 6mos. at 5 per cent?

165.45 principal.
9
6)14.8905 Int. at 6 per cent.
2.4817$\frac{1}{2}$ subtracted.
Ans. $12.4088 Int. at 5 per cent.

2. What is the interest of 5 dolls. 93cts. for 2 years and 8 months, at 3 per cent?
Ans. 47cts. 4m.

3. What is the interest of 45 dolls. for 6 months, at 8 per cent?
Ans. 1doll. 80cts.

Case V.

To find the interest of any sum by decimals.

RULE.—Multiply the principal by the ratio, and that product by the time; the last product will be the interest required.

Note. The ratio is the simple interest of $1 for 1 year at the rate agreed on, thus

At 3 per cent. the ratio is .03
At 4 " " " is .04
At 5 " " " is .05
At 5½ " " " is .055
At 6 " " " is .06
At 6½ " " " is .065
At 7 " " " is .07
At 8 " " " is .08, &c.

Examples.

1. What is the interest of $23 23cts. for 3 years at 5½ per cent?

```
     23.23
      .055 ratio.
    ------
     11615
    11615
    ------
   1.27765
         3 time.
    ------
  $3.83295 Ans.
```

2. What is the interest of $223.14 for 5 years, at 6 per cent? Ans. $66.942.

3. What is the interest of $10.15 for 12 years, at 3 per cent? $3.654.

4. What is the amount of 12½ cents, for 500 years, at 6 per cent? Ans. $3.875.

Case VI.

To compute interest on Notes, Bonds, &c.

Principles.*—1. If the contract be for the payment of interest annually, the interest becomes due at the end of each year, and if it

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* These are the principles on which interest is allowed by the courts of law in Vermont. The following methods are sometimes practised in casting the interest on notes, &c.

1. *Find the amount of the principal for the whole time, and also the amount of the endorsements from the time they were made; deduct the latter from the former, and the remainder will be the sum due.* But this method would be unjust; for, suppose a note be given for $100 with interest, and $6 be paid at the end of each year for 4 years, which is endorsed on the note. Now the interest of the principal for this time is $24, just equal to the sum of the payments; but by this method the several payments all draw interest from the times they are made, the first 3 years, the second 2, and the third 1,=1 08+.72+ 36= $2.16, which goes towards paying the principal, and in this way any debt would in time be extinguished by the payment of the interest annually.

2. *Cast the interest up to the first payment, and if the payment exceed the interest, deduct the excess from the principal, and cast the interest on the remainder up to the second payment, and so on. If the payment be less than the interest, place it by itself, and cast the interest up to the next payment, and so on till the payments exceed the interests, then deduct the excess from the principal, and proceed as before.* By this method the interest is supposed to be always due at the time the payment is made. The impropriety of this, as a general rule, may be shown by an example. Supposing A has a note against B for $10000 with interest, payable in one year, and B pays $200 at the end of every 2

be not extinguished by payment, interest is to be cast upon that interest from the time it becomes due, up to the time of payment.

2. If the contract be for a sum payable at a specified time with interest, no interest becomes due till the time of payment arrives, and endorsements made before that time are to be applied exclusively to the principal.

Rule 1.

When the contract is for the payment of interest annually, and no payments have been made, find the interest of the principal for each year, separately, up to the time of payment; then find the interest of these interests, severally, from the time they become due up to the time of payment, and the sum of *all* the interests added to the principal will be the amount: *but if payments have been made,* find the amount of the principal, and also the amount of the payments to the end of the first year; subtract the latter amount from the former, and the remainder will be the principal for the second year; proceed in the same way from year to year up to the time of payment.*

Examples.

1. A's note to B for $100, with interest annually, at 6 per cent was dated January 1, 1820; what was due, principal and interest, January 1, 1824?

1st year.	$100×6=$6 Int.	
2 "	100×6= 6 "	6×18=1.08
3 "	100×6= 6 "	6×12= .72
4 "	100×6= 6 "	6× 6= .36
Principal. 100.	$24 Int.	$2.16 Int.
Int. of prin. 24.		
Int. of int. 2.16		
Amount. $126.16 Ans.		

At the end of the first year, one year's interest =$6, is due, but as it is not paid, it draws interest for the three following years=$1.08. At the end of the second year, another year's interest is due, which draws interest for 2 years; and so on.

---

months for the year, which is endorsed on the note. Now if the interest be cast by the above method, there will be found due at the end of the year, $9384.798. But this is $14.798 more than is justly due, as may be thus shown: it is plain that neither the 10000 dollars, nor the interest of it, is due till the end of the year, when their amount is 10600 dollars; B is therefore at liberty to pay or not, before that time. Now suppose B keep back these several payments, and put them to interest till the end of the year; the first will amount to 210 dollars, the second to 208, the third to 206, the fourth to 204, the fifth to 202, and the sixth to 200; and their whole amount is (210+208+206+204+202+200)=1230, so B will have 1230 dollars at the end of the year towards extinguishing the amount of the debt, and $10600—1230=9370, the sum justly due, which is $14,798 less than the former. This method allows compound interest, both upon the principal and payments, and they are compounded, that is, the interest becomes a part of the principal as often as the payments are made.

* It will sometimes happen that when a note has endorsements, there will be years in which no payments are made; for which years the interest is to be found by the former part of the rule; and also when the amount of the payment is less than the interest of the principal, subtract the amount from that interest, and find the amount of the remainder up to the final payment.

2. B's note to C for $50, with interest annually, was dated Nov. 20, 1822, on the back of which were the following endorsements, viz. May 20, 1823, received $14, and Feb. 26, 1824, $30; what was due January 2, 1825?

```
 Prin. $50    Pay't. $14    Prin. $38.58    Pay't. $30
        6             3                6           4.4    Prin. 9.574
     ----          ----          -------        ------             .7
 Int. 3.00          .42           2.3148         1.320       --------
       50          14              38.58         30.           .067018
     ----          ----          -------        ------          9.574
Am't. 53.    Am't. 14.42    Am't. 40.894    Am't. 31.32      --------
      14.42                        31 32                 Ans. $9.641
     ----                        -------              due Jan. 2, 1825.
2 prin.38.58                3d prin. 9.574
```

3. D's note to E for $1000, with interest annually, was dated May 5, 1822, on which the following payments were made; viz. Nov. 17, 1822, $300; April 23, 1823, $50, and Aug. 11, 1823, $520; what was due June 5, 1824? Ans. $201.713.

4. C's note to D for $200, with interest annually, was dated June 15, 1821, on the back of which was endorsed, Sept. 15, 1821, $4, and Jan. 21, 1823, $15; what was due Jan. 15, 1824? Ans. $217.196.

Rule 2.

When the contract is for a sum payable at a specified time, with interest, and payments are made before the debt becomes due; find the interest of the principal up to the first payment, and set it aside; subtract the payment from the principal, and find the interest of the remainder up to the next payment, which interest set aside with the former, and so on up to the time the debt becomes due; and the sum of the interests added to the last principal, will be the amount due at that time; after the debt falls due, the interest is to be extinguished annually, if the payments are sufficient for that purpose.

Examples.

1. E's note to F for $75.25, payable in 2 years, with interest, was dated May 1, 1822, on which was endorsed Jan. 13, 1823, $25.25; what was due May 1, 1824?

```
          year. mo. day.
          1823   0   13      1st prin. 75.25 x 4.2=$3.16 interest.
          1822   4    1         Pay't. 25.25
          --------------               -----
1st time.        8   12      2d prin. 50.00 x 7.8=  3.90 interest.
                                       7.06         ----
          1824   4    1                -----        7.06 interests.
          1823   0   13        Ans. $57.06
          --------------
2d time.   1     3   18
```

2. F gave his note to G for $5000 with interest, dated Sept. 1, 1820, and payable January 1, 1824; on the 13th of June, 1822, he paid $2500, and Aug 25, 1823, $2500 more; what was due when the time of payment arrived?
Ans. $715.

3. G's note of $365.37 was dated Dec. 3, 1817, payable Sept. 11, 1820; June 7, 1820, he paid $97.16; what was due when the time of payment arrived?
Ans. $327.46.

QUESTIONS.

1. What is Interest?
2. How is it computed?
3. What is understood by the principal?
4. What is the rate?
5. What is the amount?
6. Of how many kinds is interest?
7. What is Simple Interest?
8. How do you find the interest on any sum in Federal or English money?
9. What is Case II.?
10. What is the rule?
11. What is Case III.?
12. What is the rule?
13. What is Case IV.?
14. What is the rule?
15. What is Case V.?
16. What is the rule?
17. What is Case VI.?
18. What is the first principle?
19. What the second?
20. What is the first rule?
21. What the second?

2. Compound Interest.

Compound Interest is that which arises from making the interest a part of the principal at the end of each year, or stated time for the interest to become due.

Rule I.

Find the amount of the given principal for the first year, or to the first stated time for the interest to become due, by simple interest, and make the amount the principal for the next year, or stated period; and so on to the last. From the last amount subtract the given principal, and the remainder will be the compound interest required.

Examples.

1. What is the compound interest of $125 for 2 years and 6 months,* at 6 per cent?

$125	
6	
7.50	Int. for 1st year.
125.	Prin. added.
132.50	Amount for 1 year.
6	
7.9500	Int. for 2d year.
132.50	Prin. added.
140.45	Amount for 2d year.
3	
4.2135	Interest for 6 months.
140.45	Principal added.
144.6635	Amount for 2 ys. 6 mo.
125.	1st Prin. subtracted.
$19.663	Com. Int. required.

2. What is the compound interest of $100 for 4 years, at 6 per cent? Ans. $26.246.

3. What is the compound interest of $200 for 1 year, at 6 per cent, interest due every 4 months? Ans. $12.241.

4. What is the amount of $236 at 6 per cent, compound interest, for 3 years, 5 months and 6 days? Ans. $288.387.

5. What is the amount of $150 at 6 per cent, compound interest, for 2 years, the interest becoming due at the end of every six months? Ans. $168.826.

6. What is the compound interest of $768 for 4 years, at 6 per cent? Ans. $201.58.

7. What is the compound interest of $560 for 3 years and 6 months, at 6 per cent? Ans. $126.977.

Rule 2.

By Decimals.

1. Find the amount of 1 dollar for 1 year at the given rate, and multiply this amount as many times into itself as the whole number of years, less by 1.

2. Multiply the last product by the principal, and the product will be the amount for the time; from which subtract the principal, and the remainder will be the interest required.

---

* When there are months and days, first find the amount for the years, or stated periods. then find the amount of this amount for the months and days at simple interest. Any sum doubles at 6 per cent, compound interest, where the interest becomes due at the end of each year, in 11 years, 10 months and 22 days, and at simple interest in $16\frac{6}{10}$ years.

Examples.

1. What is the compound interest of $500 for 3 years, at 5 per cent?

```
  1.05 am't. of $1 for 1 year.
  1.05
   525
 1050
1.1025 once into itself.
  1.05
 55125
110250
1.157625 twice into itself.
     500 principal.
578.812500 amount.
500
$78.812 interest required.
```

2. What is the compound interest of $125 for 2 years, at 6 per cent? Ans. $15.45.

3. What is the amount of 760 dollars 50 cents, for 4 years, at 4 per cent?

Ans. $889.677.

4. What is the amount of $6.66 for 2 years, at 9 per cent?

Ans. $7.912.

5. What is the amount of £720 for 4 years, at 5 per cent per annum? Ans. £875 3s. 3¼d.

QUESTIONS.

1. What is Compound Interest?
2. What is the rule for finding Compound Interest?
3. What is the rule for Compound Interest by decimals?

---

§ Discount.

Discount is an allowance made for the payment of money before it becomes due, and is the difference between that sum, due some time hence, and its present worth.

The present worth of any sum, or debt due some time hence, is such a sum as would, in the given time, at the given rate, if put to interest, amount to the sum or debt then due.*

* It is very evident that an allowance ought to be made for the payment of money before it becomes due, which is supposed to bear no interest till after it is due; for it is plain that the debtor, by keeping the money in his own hands, could derive advantage from putting it to interest for that time, but by paying it before it is due, he gives that advantage to another. And hence some debtors will be ready to say, that since by not paying the money till it becomes due, they may employ it at interest; therefore, by paying it before it is due, they shall lose that interest, and for that reason, all such interest ought to be discounted. But this is not true, for they cannot be said to lose the interest till the time the debt becomes due; whereas we are to consider what is at present lost by paying a debt due some time hence. Now the present worth of $106, due one year hence, discounting at 6 per cent, is evidently $100; for $100 put to interest, will amount to $106 at the end of the year, and just pay the debt, so that a debt of $106, due one year hence, discounting at 6 per cent, is justly satisfied by the present payment of $100. But the interest of $106, the time and rate as above, is $6.36, which exceeds the discount 36 cts. equal to the interest upon the discount for that time. The discount, therefore, of any sum, payable at some future time, is a sum, which put to interest for the given time and rate, will amount precisely to the interest on the given sum for that time.

Case I.

To find the present worth of a sum due some time hence.

Rule.—As the amount of 100 dollars for the given time and rate, is to 100, so is the given sum to its present worth.

Examples.

1. What is the present worth of 125 dollars, due 3 years hence, discounting at the rate of 6 per cent?

```
                  doll.
$100  Then 118 : 100 : : 125
  18              125
-----          -------     $
1800 Int.  118)12500(105.932 12/59
 100            118       (Ans.
-----           -----(pr't worth.
 118 Am't.        700
                  590
                  ----
                  1100
                  1062
                  ----
                   380
                   354
                   ---
                    260
                    236
                    ---
                     24
```

2. What is the present worth of 376 dolls. 20 cts. due at the end of 1 year and 6 months, discounting at 5 per cent?

Ans. $350.

3. A minister, settled with a salary of 300 dollars a year, wishing to build a house, his parishioners agreed to pay him 4 year's salary in advance, discounting at 6 per cent. per annum; how much ready money must they pay?

Ans. $1047 04.

4. What is the present worth of £150, payable in 3 months, discounting at 5 per cent?

Ans. £148 2s. 11½d.

Case II.

To find the discount of any sum due some time hence.

Rule.—Find the present worth and subtract it from the given sum; or say, as the amount of 100 dollars for the given time and rate, is to the interest of 100 dollars for the given time and rate, so is the given sum to the discount required.

Examples.

1. What is the discount upon 125 dollars, due 3 years hence, at 6 per cent?

Ans. 19.067\frac{47}{59}$ discount.

```
125 given sum.
105.932 12/59 present worth.
--------------
 19.067 47/59 discount, or
        118 : 18 : : 125
```

2. What is the discount upon 560 dollars due 9 months hence, at 8 per cent?

Ans. 31.69\frac{43}{53}$.

3. What is the discount of 50 dollars, due 2 years hence at 12 per cent?

Ans. $9.677.

QUESTIONS.

1. What is Discount?
2. What is the present worth of a sum due some time hence?
3. How is the present worth found?
4. How is the discount found?

¶. Commission, Brokerage and Insurance.

1. Commission is an allowance of so much per cent, to an agent abroad, for buying and selling goods for his employer.

2. Brokerage is an allowance of so much per cent. to a person called a Broker, for assisting merchants and others in procuring and disposing of their goods, &c.

3. Insurance is a premium of so much per cent. given to certain persons, or companies, for a security for making good the loss of ships, buildings, goods, &c. which may happen by fire, storms, &c.*

Rule.

Commission, Brokerage and Insurance are calculated by the fir rule given for computing Simple Interest.

Examples.

1. What must I allow for selling 525 dollars worth of Goods, at 3 per cent. commission?

$$\begin{array}{r} 525 \\ 3 \\ \hline 15.75 \end{array}$$ Ans.

2. What comes the commission of 827 dolls. 64 cts. to, at $2\frac{1}{2}$ per cent?
Ans. 20 dolls. 69 cts. 1 m.

3. What is the brokerage of £610, at $\frac{1}{4}$ per cent?
Ans. £1 10 s. 6 d.

4. If I allow my Broker $3\frac{1}{2}$ per cent. what must I allow him for purchasing 2525 dolls. worth of goods?
Ans. $88 37 cts. 5 m.

5. What is the insurance of a house, worth 3460 dollars, at $\frac{1}{2}$ per cent? Ans. $17 30 cts.

6. What is the insurance of £1200, at $7\frac{1}{2}$ per cent?
Ans. £90.

QUESTIONS.

1. What is Commission?
2. What is Brokerage?
3. What is Insurance?
4. What is the rule for computing Commission, Brokerage, and Insurance?

---

* Insurance is made by a writing called a *policy*, which should always be sufficient to cover the principal and premium; that is, a policy to secure the payment of 100 dollars, at 6 per cent. must be taken out for 106 dollars.

SECTION VI.

I. Practice.*

Practice is a contraction of the Rule of Three when the first term is a unit. It took its name from its daily use among merchants. The necessity of this rule is nearly superceded by the use of Federal Money.

Rule.

1. Suppose the price of the given quantity to be £1, 1 s. 1 d. or 1 qr. per pound, yard, &c. as is most convenient, then the quantity itself will be the answer at the supposed price.

2. Divide the given price into aliquot parts, either of the supposed price, or of one another, and the sum of the quotients belonging to each, will be the true answer required.

Proof.—By the Rule of Three.

Table of aliquot parts of Money.

Parts of shill.	of a pound.		Parts of pound.		Parts of a pound.	
d.	s.	£	s. d.	£	d. q.	£
6	$\frac{1}{2}$	$\frac{1}{40}$	10 0	$\frac{1}{2}$	10 0	$\frac{1}{24}$
4	$\frac{1}{3}$	$\frac{1}{60}$	6 8	$\frac{1}{3}$	8 0	$\frac{1}{30}$
3	$\frac{1}{4}$	$\frac{1}{80}$	5 0	$\frac{1}{4}$	5 0	$\frac{1}{48}$
2	$\frac{1}{6}$	$\frac{1}{120}$	4 0	$\frac{1}{5}$	2 2	$\frac{1}{96}$
1½	$\frac{1}{8}$	$\frac{1}{160}$	3 4	$\frac{1}{6}$	Parts of a penny.	
1	$\frac{1}{12}$	$\frac{1}{240}$	2 6	$\frac{1}{8}$	qr.	d.
¾	$\frac{1}{16}$	$\frac{1}{320}$	1 8	$\frac{1}{12}$	1	$\frac{1}{4}$
½	$\frac{1}{24}$	$\frac{1}{480}$	1 0	$\frac{1}{20}$	2	$\frac{1}{2}$
¼	$\frac{1}{48}$	$\frac{1}{960}$			3	$\frac{3}{4}$

---

* Practice admits of a great number of cases. But as the rule is losing its importance, and going out of use in consequence of the introduction of Federal Money, it was thought best to introduce one general rule, and not perplex the scholar with a multiplicity of almost useless cases. This rule, with a little attention, will readily be applied to the solution of all questions which belong to it.

When there is a fractional part of a pound or yard, take an equal part of the price of one yard, that is, for one half, take half the price; for one fourth, take one fourth; for three fourths, take three fourths of the price of one pound or yard.

Examples.

1. What will 225 yards cost, at 2 qrs. per yard?

2)225d. the price at 1d. per yard.

12)112½d. the price at 2 qrs. because 2 qrs. is ½ of 1d.

9s. 4½d. Ans.

2. What will 1776 yards cost, at 3d. per yard? Ans. £22 4s.

3. What will 263 yards cost, at 3 qrs. per yard? Ans. 16s. 5¼d.

4. What will 135 yards cost, at ½d. per yard? Ans. 5s. 7½d.

5. What will 937½ yards cost, at £3 17s. 8d. per yard? Ans. £3640 12s. 6d.

6. What will 784 yards cost, at 4d. per yard? Ans. £13 1s. 4d.

7. What will 395 gallons cost, at 4s. 6d. per gallon? Ans. £88 17s. 6d.

8. What will 426 lb. cost, at 11d. per pound? Ans. £19 10s. 6d.

9. What will 354½ yards cost, at 1 qr. yard? Ans. 7s. 4d. 2½qrs.

10. What will 845 yards cost, at 8s. per yard? Ans. £338.

11. What will 843 yards cost, at 6s. 8d. per yard? Ans. £281.

12. What will 468 lb. cost, at 6d. per pound? Ans. £11 14s.

13. What will 5275 lb. cost, at 2d. per pound? Ans. £43 19s. 2d.

14. What will 435 lb. cost, at 4½d. per pound? Ans. £8 3s. 1½d.

15. What will 426 yards cost, at 4s. 9d. per yard? Ans. £101 3s. 6d.

16. What will 204 yards cost, at 1s. 1d. per yard? Ans. £11 1s.

17. What will 568¼ yards cost at 7d. per yard? Ans. £16 11s. 5¾d.

18. What will 68 lb. cost, at 4s. 6d. per pound? Ans. £15 6s.

19. What will 76 yards cost, at 2d. per yard? Ans. 12. 8d.

QUESTIONS.

1. What is Practice?
2. Why is it so called?
3. Is an acquaintance with this rule as necessary as it was formerly?
4. For what reason?
5. What is the rule for Practice?
6. What is the method of proof?

2. Tare and Trett.

TARE AND TRETT are practical for deducting certain allowances which are made by merchants and tradesmen in selling their goods by weight.

Gross weight is the whole weight of any sort of goods, together with the box, cask, or bag, &c. that contains them.

Tare is an allowance to the buyer, for the weight of the box, cask, or bag, &c. which contains the goods bought.

Trett is an allowance of 4 lb. in every 104 lb. for waste, dust, &c.

Cloff is an allowance of 2 lb. on every 3 cwt.

Suttle is the weight when part of the allowance is deducted from the gross.

Net weight is what remains after the allowances are made.

Case I.

When Tare is so much per box, cask, &c.

RULE.—Multiply the number of boxes, &c. by the tare, subtract it from the gross, and the remainder will be the net weight.

Examples.

1. In 5 hogsheads of sugar, each weighing 8 cwt. 1 qr. 9 lb. gross, tare 24 lb. per hogshead; how much net weight?

	cwt.	qr.	lb.	
24×5=	1	0	8	
	8	1	9	
			5	
	41	2	17	gross.
	1	0	8	tare.
Ans.	40	2	9	net.

2. In 241 barrels of figs, each 3 qrs. 19 lb. gross, tare 10 lb. per barrel, how many pounds net?
Ans. 22413.

3. What is the net weight of 14 hogsheads of tobacco, each 5 cwt. 2 qrs. 17 lb. gross, tare 100 lb. per hogshead?
Ans. 66 cwt. 2 qrs. 14 lb.

Case II.

When Tare is so much per cwt.

RULE.—Divide the gross weight by the aliquot parts of a cwt. contained in the tare, and subtract the quotient from the gross, the remainder is the net weight.

Examples.

1. What is the net weight of 33 cwt. 2 qrs. 18 lb. gross, tare 16 lb. per cwt.?

	cwt.	qr.	lb.	
16 lb.=$\frac{1}{7}$)	33	2	18	gross.
	4	3	$6\frac{4}{7}$	
Ans.	28	3	$11\frac{3}{7}$	net.

2. In 25 barrels of figs, each 2 cwt. 1 qr. gross, tare 8 lb. per cwt. how much net?
Ans. 52 cwt. 0 qrs. 26 lb.

Case III.

When Trett is allowed with Tare.

RULE.*—Divide the suttle weight by 26, and the quotient is the trett; subtract the trett from the suttle, and the remainder is the net weight.

Examples.

1. In 9 cwt. 2 qrs. 17 lb. gross, tare 37 lb. and trett as usual, how much net?

	cwt.	qr.	lb.	
	9	2	17	gross.
		1	9	Tare.
26)	9	1	8	suttle.
		1	11	trett.
Ans.	8	3	25	net.

2. In 342 cwt. 2 qrs. 14 lb. gross, tare 16 lb. per cwt. and trett as usual, how much net?
Ans. 282 cwt. 1 qr. $14\frac{12}{13}$ lb.

3. In 7 casks of prunes, each weighing 3 cwt. 1 qr. 5 lb. gross, tare $17\frac{1}{2}$ lb. per cwt. and trett as usual, how much net?
Ans. 18 cwt. 2 qrs. 26 lb.

QUESTIONS.

1. What are Tare and Trett?
2. What is gross weight?
3. What is Tare?
4. What is Trett?
5. What is Cloff?
6. What is Suttle?
7. What is Net weight?
8. How do you proceed when the Tare is so much per box, &c.
9. How, when Tare is so much per hundred weight?
10. How, when Trett is allowed with Tare?

8. Equation of Payments.

EQUATION OF PAYMENTS teaches to find the time for paying at once several debts due at different times, so that no loss shall be sustained by either party.

RULE.†—Multiply each payment by the time at which it is due, and divide the sum of the products by the sum of the payment, and the quotient will be the time required.

* You divide by 26, because the trett is one twenty-sixth. When cloff is allowed, after deducting the tare and trett, divide the suttle by 168, and the quotient is the cloff which subtract from the suttle, and the remainder is the net weight. You divide by 168, because the cloff is 1 lb. in every 168 lb. or $\frac{1}{168}$.

† This rule supposes that there is just as much gained by keeping some of the debts after they are due, as is lost by paying the others before they are due. But this is not exactly true: for by keeping a debt unpaid after it is due, there is gained the interest of it for that time; but by paying a debt before it is due, the payer loses only the discount, which is somewhat less than the interest, as has already been shown. The rule, however, is sufficiently correct for practical purposes.

Examples.

1. A owes B \$750, to be paid as follows, viz. \$500 in 2 months, \$150 in 3 months, and \$100 in $4\frac{1}{2}$ months; what is the equated time to pay the whole?

$500 \times 2 = 1000$
$150 \times 3 = 450$
$100 \times 4.5 = 450$

mo.
$750\,)\,1900\,(\,2\frac{400}{750} = 2\frac{7}{15}$ Ans.
1500
400

2. B owes C \$190, to be paid as follows, viz. \$50 in 6 months, \$60 in 7 months, and \$80 in 10 months; what is the equated time to pay the whole? Ans. 8 months.

3. C owes D a certain sum of money, which is to be paid $\frac{1}{2}$ in 2 months, $\frac{1}{3}$ in 4 months, and the remainder in 10 months; what is the equated time to pay the whole? Ans. 4 months.

QUESTIONS.

1. What does the Equation of Payments teach?
2. What is the rule?
3. Is the rule perfectly correct?
4. Why is it not?
5. Why then is it introduced?

II. Fellowship.

FELLOWSHIP is a general rule, by which merchants and others, trading in company, with a joint stock, compute each person's particular share of the gain or loss.

Fellowship is of two kinds, *Single* and *Double.*

1. *SINGLE FELLOWSHIP.*

Single Fellowship is when the stocks or times are equal.

Rule.

If the stocks are equal, say, as the whole time is to the whole gain or loss, so is the time each man's stock is employed to his share of the gain or loss; and *if the times are equal,* say, as the sum of the stocks is to the whole gain or loss, so is each man's share in the stock to his share in the gain or loss.

Proof.

Add all the shares of the gain or loss together, and the sum will equal the whole gain or loss, if the work be right.

1. A and B made a joint stock of $500, of which A put in $350, and B $150, they gain $75; what is each man's share of the gain?

$ 500 : $ 75 :: { $ 350 : Ans. 52.50 A's share.
150 : 22.50 B's share.

75.00 proof.

2. A, B and C companied; A put in £480, B £680, C £840, and they gained £1010; what is each man's share?

£	s.		
242	8	A's.	Ans.*
343	8	B's.	
424	4	C's.	

3. Three persons make a joint stock, of which each puts in an equal share; A continues his stock in trade 4 months, B his 6 months, and C his 10 months, and they gained $480; what was each man's share?

$96	A's	Ans.
144	B's	
240	C's	

480 proof.

4. Divide $160 among 4 men, so that their shares shall be as 1, 2, 3, and 4.

Ans. { 16, 32, 48, 64

160 proof.

5. A person dying, bequeathed his estate to his 3 sons; to the eldest he gave $560, to the next, $500, and to the other $450; but when his debts were paid, there were only $950 left; what was each son's share?

$		
352.317+	1st	Ans.
314.569+	2d	
283.112+	3d	

6. D and E companied; D put in $125, and took out $\frac{4}{15}$ of the gain; what did E put in?

Ans. $375.

2. DOUBLE FELLOWSHIP.

Double Fellowship is when unequal stocks are employed for unequal times.

RULE.—Multiply each man's stock by the time of its continuance in trade; then, as the sum of the products is to the whole gain or loss, so is each product to its share of the gain or loss.

* The shares of gain or loss are evidently as the stocks when the times are equal, so when the stocks are equal, the shares are evidently as the times; wherefore, when neither the stocks nor times are equal, the shares must be as their product.

Examples.

1. Three farmers hired a pasture for $60.50. A put in 5 cows for 4½ months, B put in 8 for 5 months, and C put in 9 for 6½ months; how much must each pay of the rent?

$5 \times 4.5 = 22.5$
$8 \times 5 = 40$
$9 \times 6.5 = 58.5$
121.

121 : 60.50 :: 22.5
22.5
30250
12100
12100
121)1361.250(11.25 A's. $
121
151
121
302
242
605
605

121 : 60.50 :: 40
40
121)242000(20 B's. $
242
0000

121 : 60.50 :: 58.5
58.5
30250
48400
30250
121)3539250(29.25 C's.
242
1119
1089
302
242
605
605

$11.25 A's.
20.00 B's. } Ans.
29.25 C's.
60.50 proof.

2. Two merchants entered into partnership for 18 months; A at first put in £100, and at the end of 8 months put in £50 more; B at first put in £275, and at the end of 4 months, took out £70; at the end of the 18 months they had gained £263; what is each man's share?

£	s.	d.		
96	9	6 $\frac{42}{627}$	A's.	Ans.
166	10	5 $\frac{585}{627}$	B's.	
263	0	0	proof.	

3. Three men hire a pasture for 100 dollars; A puts in 40 oxen for 20 days, B 30 oxen for 40 days, and C 50 oxen for 10 days; how much must each man pay?

$
32 A's.
48 B's. } Ans.
20 C's.
100 proof.

QUESTIONS.

1. What is Fellowship?
2. Of how many kinds is Fellowship?
3. What is Single Fellowship?
4. What is the rule for Single Fellowship?
5. What is Double Fellowship?
6. What is the rule for Double Fellowship?

5. Barter.

BARTER is the exchanging of one quantity for another, and teaches merchants so to proportion their quantities that neither shall sustain loss.

Case I.

When the quantity of one commodity is given, with its value, or the value of its integers, and also the value of the integer of some other commodity to be exchanged for it, to find the quantity of this commodity.

RULE.—Find the value of the given quantity, then find how much of the other, at the rate proposed, may be had for the same sum.

Examples.

1. A has 350 yards of cloth at 1s. 4d. per yard, which he would exchange with B for sugar at 25s 6d per cwt.; how much sugar will the cloth come to?

350yds. at 1s. 4d.=466s. 8d.= 5600d. and 25. 6d.=306d.

d. cwt. d.
Then 306 : 1 : : 5600
cwt. qr. lb.
Ans. 18 1 $5\frac{2}{3}$ nearly.

2. A has $7\frac{1}{2}$ cwt. of sugar at 8d per pound, for which B gave him $12\frac{1}{2}$ cwt. of flour; what was the flour per pound?

Ans. $4\frac{4}{5}$d.

3. How much tea at 9s. 4d. per pound, must be given in barter for 156 gallons of wine, at 12s. $3\frac{1}{2}$d. per gallon?

Ans. 205lb. 13$\frac{5}{14}$oz.

Case II.

When the quantities of two commodities are given, and the rate of selling them, to find, in case of inequality, how much of some other commodity, or how much money should be given.

RULE.—Find the separate values of the two given commodities and their difference will be the balance, or value of the other commodity.

Examples.

1. A has 30 cwt. of cheese at $3.927 per cwt. which he barters with B for 9 pieces of broadcloth at $12.50 per piece; which must receive money, and how much?

Ans. B must pay A $5.31.

2. I have 63 gal. of molasses at $62\frac{1}{2}$ cts. per gal. which I would exchange for 52 bushels of rye at 75cts. per bushel; must I pay or receive money, and how much?

Ans. I must receive $37\frac{1}{2}$cts.

Case III.

When one commodity is rated above the ready money price, to find the bartering price of the other.

RULE.—As the ready money price of the one is to its bartering price, so is that of the other to its bartering price.

Examples.

1. A and B barter; A has 150 gallons of brandy at $1.20 per gal. ready money, but in barter, would have $1.40; B has linen at 60cts. per yard, ready money; how ought the linen to be rated in barter, and how many yards are equal to A's brandy?

Ans. barter price, 70cts. and B must give A 300 yds.

2. A has coffee which he barters with B at 10d. per pound more than it cost him, against tea, which stands B in 10s. the pound, but puts it at 12s. 6d. I would know how much the coffee cost at first. Ans. 3s. 4d.

3. B delivered 3 hhds. of brandy at 6s. 8. per gallon to C for 126 yards of cloth, what was the cloth per yard?

Ans. 17s. 6d.

4. C has tea at 78cts. per lb. ready money, but in barter, would have 93cts.; D has shoes at 7s. 6d. per pair, ready money; how ought they to be rated in barter, in exchange for tea?

Ans. $1.49.

5. C has candles at 6s. per dozen, ready money; but in barter he will have 6s. 6d. per dozen; D has cotton at 9d. per lb. ready money, what price must the cotton be at in barter, and how much cotton must be bartered for 100 dozen of candles?

Ans. the cotton 9¾d. per lb. in barter, and 7cwt. 0qrs. 16lb of cotton must be given for 100 doz. of candles.

QUESTIONS.

1. What is Barter?
2. What does it teach?
3. When the quantity of one of the commodities is given, how is the quantity of the other found?
4. When the quantities of both commodities are given, in case of inequality, how is it found?
5. When one of the commodities is rated above the ready money price, how is the bartering price of the other found?
6. What is the method of proof?

§. Loss and Gain.

Loss and Gain is a rule which enables merchants to ascertain the profit, or loss, in buying and selling their goods, and also teaches them how much to raise or fall in the price, in order to gain or lose so much per cent.

Case I.

To know what is gained or lost, per cent.

Rule.—Find the whole gain or loss by subtraction; then as the price it cost is to the whole gain or loss, so is $100 to the gain or loss per cent.

Examples.

1. If I buy salt for 84cts. per bushel, and sell it for $..12 per bushel, what do I gain per cent?

```
                $  cts.    $
 1.12       0.84 : 28 : : 100.00
   84             100000
 ----         ----------  $ Ans.
  .28 gain   .84)2800.000(33.333⅓
 per bush.        252
                  ---
                   280
                   252
                   ---
                    280
                    252
                    ---
                     280
                     252
                     ---
                      280
                      252
                      ---
                       28
```

2. If I buy cloth for $1.25 per yard, and sell it again for $1.375 per yard, what do I gain per cent? Ans. $12.50.

3. If I buy sugar for 6¼d per lb. and sell it for £2 3s 9d per cwt. do I gain or lose, and how much per cent?

Ans. £2. 17s. $8\frac{4}{11}$d. loss.

4. At 3d in the shilling profit, how much per cent?

Ans. £25.

Case II.

To know how a commodity must be sold to gain or lose so much per cent.

Rule.—As $100 is to the price, so is $100 with the profit added or loss subtracted to the gaining or losing price.

Examples.

1. If I buy cloth for $0.75, how must I sell it to gain 9½ per cent?

100	\$	\$
9.50	100 : .75 : :	109.50
		.75
109.50		
		54750
		76650
		100) 82.1250
		Ans. $0.821

2. If I buy cloth for $2.50 per yard, how must I sell it to lose 17½ per cent? $2.06¼.

3. If tea cost 3s. 8d. per lb. how must it be sold to gain 12½ per cent?

Ans. 4s. 1½d per lb.

4. Bought 40 gallons of rum at 75 cents per gallon, of which 10 gallons leaked out by accident; how must I sell the remainder to gain 12½ per cent on the prime cost? Ans. $1.125 per gal.

QUESTIONS.

1. What is Loss and Gain?
2. How do you proceed to find what is gained or lost per cent?
3. How do you proceed to find how a commodity must be sold to gain or lose so much per cent?
4. What is the method of proof?

7. Alligation.

Alligation teaches to mix commodities of different qualities, so that the composition may be of a middle quality.

Alligation is of two kinds, *Medial* and *Alternate.*

1. *ALLIGATION MEDIAL.*

Alligation Medial is the method of finding the rate of the compound, from having the prices and quantities of the several commodities given.

RULE.*—Multiply each quantity by its price, and divide the sum of the products by the sum of the quantities, the quotient will be the rate of the compound required.

* The truth of this rule is too evident to need a demonstration; for multiplying each quantity by its price, and adding the products gets the price of the whole quantity; then as the sum of the quantities, or number of bushels, is (by the Rule of Three) to the whole price of the mixture, so is one bushel of the mixture to its price. But as 1 is the third term in this and all similar cases, we have only to divide the whole price by the whole quantity.

Examples.

1. If I mix 8 bushels of wheat at \$1.20 per bushel, 12 bushels of rye at 60 cents, and 10 bushels of corn at 50 cents, together; what is a bushel of the mixture worth?

1.20	60	50	8	
8	12	.10	12	
—	—	—	10	
9.60	7.20	5.00	—	
7.20			30	sum of the quantities.
5.00				
—				
21.80 sum of prod.				

Then 30)21.80(72$\frac{2}{3}$ per bush. Ans.
210
—
80
60
—
20

2. A merchant mixed 6 gallons of wine at 4s. 10d. a gallon, with 12 gallons at 5s. 6. and 8 at 6s. 3½d. a gallon; what is a gallon of the mixture worth?

Ans. 5s. 7d.

3. If 5lb. of tea at 6s. per lb. 8lb. at 5s. and 4lb. at 4s. 6d. be mixed together, what is a pound of the mixture worth?

Ans. 5s. 2$\frac{2}{17}$d.

4. A goldsmith melted together 10 oz. of gold 20 carats fine, 8 oz. 22 carats fine, and 1 lb. 8 oz. 21 carats fine; what is the fineness of the mixture?

Ans. 20$\frac{18}{19}$ carats fine.

2. ALLIGATION ALTERNATE.

Alligation Alternate is the method of finding what quantity of each of the articles, whose rates are given, will compose a mixture of a given rate.

Alligation Alternate is the reverse of Alligation Medial, and may be proved by it.

Rule 1.*

1. Write the rates of the several articles in a column under each other, beginning with the least, and place the rate of the compound at the left hand.

2. Connect each rate which is less than that of the compound with one that is greater, and each that is greater with one that is less.

3. Write the difference between each rate and that of the compound against the number with which each is connected.

4. Then if only one difference stand against any rate, it will be the quantity belonging to that rate; but if there be more than one, their sum will be the quantity.

* As a less rate is in all cases connected with a greater, and the differences between them and the mean rate placed alternately, there is just as much gained by one quantity as there is lost by the other. Thus in getting answer first,

Examples.

1. A farmer wishes to mix rye at 4s. corn at 3s. barley at 2s. 6d. and oats at 2s per bushel, so that the mixture may be worth 2s. 10d. per bushel; how much of each sort must he take?

		bush.	
34	24	14 oats.	1 Ans.
	30	2 barley.	
	36	4 corn.	
	48	10 rye.	

		bush.	
or 34	24	2 oats.	2 An.
	30	14 barley.	
	36	10 corn.	
	48	4 rye.	

			bush.	
or 34	24	14	14 oats.	3 Ans.
	30	2+14	16 barley.	
	36	4	4 corn.	
	48	4+10	14 rye.	

			bush.	
or 34	24	2+14	16 oats.	4 Ans.
	30	2	2 barley.	
	36	10+4	14 corn.	
	48	10	10 rye.	

			bush.	
or 34	24	2+14	16 oats.	5 Ans.
	30	2+14	16 barley.	
	36	4+10	14 corn.	
	48	4+10	14 rye.	

			bush.	
or 34	24	2	2 oats.	6 Ans.
	30	2+14	16 barley.	
	36	10+4	14 corn.	
	48	4	4 rye.	

			bush.	
or 34	24	2+14	16 oats.	7 Ans.
	30	14	14 barley.	
	36	10	10 corn.	
	48	10+4	14 rye.	

Proof to 1st Ans.

$$\begin{array}{r} 24\times 14=336 \\ 30\times 2=60 \\ 36\times 4=144 \\ 48\times 10=480 \\ \hline 30)1020(34d. \\ 90 \\ \hline 120 \\ 120 \\ \hline \end{array}$$

All the other answers may be proved to be correct, in the same way.

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example first, 24 and 30, which are below the mean, 34, are linked to 48 and 36, which are above the mean, and the difference, 10, between 24 and 34, is placed against 48, and 14, the difference between 34 and 48, is placed against

2. A merchant would mix wines at 14s. 15s. 19s. and 22s. per gallon, so that the mixture may be worth 18s. per gallon; how much must he take of each sort?

1 Ans. { 4 at 14s. / 1 at 15s. / 3 at 19s. / 4 at 22s. }   2 Ans. { 5 at 14s. / 1 at 15s. / 7 at 19s. / 4 at 22s. &c. }

Rule 2.

When the quantity of the whole composition is limited to a certain sum; find the differences by linking as before; then say, as the sum of the quantities or differences is to the given quantity, so is each of the differences to the required quantity of each rate.

Examples.

1. How much water at 0 cts. per gallon, must be mixed with brandy at $1.25 per gallon, so as to fill a vessel of 80 gallons, and that a gallon of the mixture may be worth $1?

100 { 0 —— 25 / 1.25 —— 100 }

125

gal.		gal.		gal.		gal.
125	:	80	::	25	:	16 water.
				100	:	64 brandy.
						80 given quantity.

2. How much silver of 15, of 17, of 18, and 22 carats fine, must be melted together to form a composition of 40 oz. 20 carats fine?

oz.

Ans. { 5 of 15 / 5 of 17 / 5 of 18 / 25 of 22 } car. fine.

3. A grocer would mix teas at 3s. 4s. and 4s. 6d. per pound, and would have 30 lb. of the mixture worth 3s. 6d. per lb. how much of each must he take?

lb.

Ans. { 18 at 3s. / 6 at 4s. / 6 at 4s. 6d. }

4. How many gallons of water worth 0s. per gallon, must be mixed with wine worth 3s. per gallon, so as to fill a cask of 100 gallons, and that a gallon of the mixture may be afforded at 2s. 6d.?

gall.

Ans. { $16\frac{2}{3}$ water. / $83\frac{1}{3}$ wine. }

24, and the same of 30 and 36. Now 14 bushels at 24d. is 336d. and 14 bushels at 34d. the mean rate, is 476d. and 476—336=140d. so that there is here a loss of 140d. And again, 10 bushels of rye at 48d. is 480d. and 10 bushels at 34d. is 340d. and 480—340=140d.; here there is a gain of 140d. precisely the sum that was lost by the other, so that the balance is preserved, and the same is true of the 30 and 36, or of any two numbers connected in this way, a greater with a less than the mean. Questions in this rule will admit of as many answers as there are different ways of linking the rates of the ingredients together; and after as many answers are found by linking the rates as can be, more answers may be formed by multiplying or dividing these by 2, 3, 4, &c.

Rule 3.

When one of the ingredients is limited to a certain quantity; find the differences as before; then as the difference standing against the given quantity is to the given quantity, so are the other differences severally, to the several quantities required.

Examples.

1. A grocer would mix teas at 12s. 10s. and 6s. with 20 lb. at 4s. per lb.; how much of each sort must he take to make the composition worth 8s. per pound?

$$8\left\{\begin{array}{ll} 4 & 4 \text{ against the given quantity.} \\ 6 & 2 \\ 10 & 2 \\ 12 & 4 \end{array}\right.$$

$$4 : 20 :: \left\{\begin{array}{l} 2 : 10 \text{ at } 6s. \\ 2 : 10 \text{ at } 10s. \\ 4 : 20 \text{ at } 12s. \end{array}\right\} \text{Ans.}$$

(lb. over the second column)

2. How much wine at 5s. at 5s. 6d. and 6s. per gallon, must be mixed with 3 gallons at 4s. per gallon, so that the mixture may be worth 5s. 4d. per gallon?

$$\text{Ans.}\left\{\begin{array}{l} \text{gal.} \\ 3 \text{ at } 5s. \\ 6 \text{ at } 5s.\ 6d. \\ 6 \text{ at } 6s. \end{array}\right\} \text{per gall.}$$

QUESTIONS.

1. What is Alligation?
2. Of how many kinds is it?
3. What is Alligation Medial?
4. What is the rule?
5. What is Alligation Alternate?
6. What is the rule for reckoning and linking quantities?
7. When the quantity of the whole composition is limited, what is the rule?
8. What is the rule when one of the quantities is limited?
9. How do you prove Alligation Alternate?

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* Questions are solved in the same way when several of the ingredients are limited to certain quantities, by finding first of one limit, and then of another.

The second rule in Alligation Alternate may be employed for finding the specific gravities of bodies. A curious instance of the application of this rule to the detection of fraud, is recorded of the celebrated Archimedes. Hiero, king of Syracuse, suspecting his crown, which he had ordered to be made entirely of pure gold, to be alloyed with some baser metal, employed Archimedes to ascertain the fact. The philosopher procured two other masses, the one of pure gold, and the other of silver or copper, and each of the same weight of the crown, to be examined, and by putting each of these separately into a vessel of water, he found the quantity of water expelled by each, and thus determined their specific gravity, and by that means the amount of gold, and also of alloy, in the crown. Thus, if we suppose the weight of each of the masses to be 10lb. and the water expelled by the copper or silver to be 8, that expelled by the gold to be 5, and that expelled by the crown, 7, so the rates of the simples will be 8 and 5, and that of the compound, 7. Then,

$$7\left\{\begin{array}{ll} 8 & 2 \\ 5 & 1 \end{array}\right. \quad \text{And} \quad \left.\begin{array}{l} 3 : 10 :: 2 : 6\tfrac{2}{3}\text{lb. copper.} \\ 3 : 10 :: 1 : 3\tfrac{1}{3}\text{lb. gold.} \end{array}\right\} \text{Ans.}$$

ARITHMETICK, PART II.

SECTION I.

POWERS AND ROOTS.

Involution.

Involution is the raising of powers. A power is a number produced by multiplying any given number continually by itself a certain number of times. Any number is itself called the *first power*; if it be multiplied by itself, the product is called the second power, or *square*; if this be multiplied by the first power again, the product is called the third power, or *cube*, and so on.

$3 = 3$ is the first power of 3 . . . . $= 3$
$3 \times 3 = 9$ is the second power or square of 3 . . $= 3^2$
$3 \times 3 \times 3 = 27$ is the third power or cube of 3 . . . . $= 3^3$
$3 \times 3 \times 3 \times 3 = 81$ is the fourth power or biquadrate of 3 $= 3^4$*

The small figures, ^{1}, ^{2}, ^{3}, ^{4}, placed over the 3, and used to designate the power, are called the *indices*, or *exponents*. The index of the first power is always omitted.

Examples.

1. What is the 5th power of 6?

6	
6	
36	2d power.
6	
216	3d power.
6	
1296	4th power.
6	
Ans. 7776	5th power.

2. What is the second power of 45? Ans. 2025.

3. What is the square of .25? Ans. .0625.

4. What is the square of $\frac{2}{3}$?† Ans. $\frac{4}{9}$.

---

* The index of the power is always one more than the number of multiplications performed; thus 3 multiplied 3 times by itself continually, is raised to the fourth power.

† A vulgar fraction is involved by raising both its terms to the power required. The involution of fractions diminishes their value.

QUESTIONS.

1. What is Involution?
2. What is a power?
3. What is any number itself called?
4. What is the product called, if a number be multiplied by itself?
5. If the second power be multiplied by the first, what is the product called?
6. How are powers designated?
7. Of which of the powers is the index always omitted?

2. Evolution.

EVOLUTION is the method of extracting roots. The root of any number, or power, is a number, which being multiplied by itself a certain number of times, will produce that power. Roots are denominated from the powers of which they are the root, and are called square, cube, biquadrate, or 2d, 3d, 4th root, &c. Thus 3 is the square root of 9, because 9 is the 2d power, or square of 3; 3, also, is the cube root of 27, because 27 is the 3d power, or cube of 3. Again, 2 is the 4th, or biquadrate root of 16, because 16 is the 4th power of 2, &c.

The following table exhibits the 2d, 3d, 4th, 5th, and 6th powers of the 9 digits, considered as roots or first powers.

TABLE.

Roots, or 1st powers	1	2	3	4	5	6	7	8	9
Squares, or 2d powers	1	4	9	16	25	36	49	64	81
Cubes, or 3d powers	1	8	27	64	125	216	343	512	729
Biquadrates, or 4th powers	1	16	81	256	625	1296	2401	4096	6561
Sursolids, or 5th powers	1	32	243	1024	3125	7776	16807	32768	59049
Square Cubes, or 6th pow.	1	64	729	4096	15625	46656	117649	262144	531441

The square root is denoted by the radical sign, $\surd$, placed before the power, and other roots by the same sign, with the index of the root placed over it. Thus $\sqrt{9}$ denotes the square root of 9, $\sqrt[3]{27}$ the cube root of 27; and $\sqrt[4]{16}$ the biquadrate root of 16.

Roots are also denoted by fractional indices. Thus $9^{\frac{1}{2}}$ denotes the square root of 9; $27^{\frac{1}{3}}$, the cube root of 27, and $6^{\frac{1}{4}}$ the biquadrate root of 16. The latter method of designating roots is most rational, and at present generally practised.

Although every number has a root, yet the complete root of the greatest part of numbers cannot be ascertained. The roots of all can, however, by the help of decimals, be obtained to a sufficient degree of accuracy for practical purposes.

A *power* is *complete*, when its root of the same name can be accurately extracted.

A *power* is *imperfect*, when its root cannot be accurately found, and the root of such a power is called a *surd*, or irrational quantity.

To prepare any number, or power, for extracting its root.

RULE.*—Beginning at the right hand, distinguish the given number into periods, each consisting of as many figures as are denoted by the index of the root, designating the periods by points placed over the first figures in each; by the number of periods will be shown the number of figures of which the root is to consist.

Examples.

1. Prepare 348753421 for extracting the square, cube, and biquadrate roots.

For the square root.	348753421
For the cube root.	348753421
For the biquadrate root.	348753421

2. Prepare 681012.1416 for extracting its square and cube roots.

Square.	681012.1416
Cube.	681012.141600

In preparing decimals, proceed from the separatrix towards the right hand, and if the last period happen to be incomplete, complete it by annexing ciphers.

1. *TO EXTRACT THE SQUARE ROOT.*

To extract the square root is to find the number which, multiplied into itself, will produce the given number. A square is a figure bounded by 4 equal straight lines, having 4 right angles, and its root is the length of one of their sides.

RULE.†—1. Having distinguished the given number into periods, find the root of the greatest square number in the left hand period,

* The reason of this rule will appear by considering that the product of any two numbers can have at most but just as many places of figures as there are in both the factors, and at least but one less, of which any one can satisfy himself by trial. From this fact, it is clear that a square number can have at most but twice as many places of figures as there are figures in the root, and at least but one less; and that a cube number cannot have more than three times the number of figures that there are figures in the root, and at least but two less, and so on. Example.—1 is the least possible root of a square number; $1\times1=1$, which is one less than the number of factors; $1\times1\times1=1$, two less than the number of factors, &c. Again, 10 is the least root consisting of two figures; $10\times10=100$, one less than the number of places in the factors, and $10\times10\times10=1000$, two less, &c.; and the same may be shown of the least roots consisting of 3, 4, &c. figures. Again, the greatest root consisting of one figure only, is 9; its square, or 2d power, is $9\times9=81$, consisting of just twice as many places as there are in 9, the root, and the cube of 9 is $9\times9\times9=729$, consisting of three times the number of places in the root. The same may in like manner be shown of 99, the greatest root consisting of two places, 999, the greatest consisting of three places, &c. These observations must make the reason for pointing off the number into periods, obvious, and must also make it evident, that each period will give one figure in the required root, and no more. They also show that one figure alone, or standing on the left hand of other full periods, may of itself constitute a period.

† The reason of this rule may be shown from the first example. Now if we suppose 529 to be so many square feet of boards, which we wish to lay down in the form of an exact square, it is evident that the square root of 529 will be the

and place it on the right hand of the given number, in the manner of the quotient in division, and it will be the first figure of the root required.

2. Subtract the square of the root already found, from the left hand period, and to the remainder bring down the next period for a dividend.

3. Double the root already found, for a divisor; seek how often the divisor is contained in the dividend, (excepting the right hand figure) and place the answer for the second figure of the root, and also on the right hand of the divisor; multiply the divisor by the figure in the root last found, subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.

4. Find a divisor as before, by doubling the figures in the root, and proceed as before to find the third figure in the root, and so on through all the periods.

PROOF.—Multiply the root by itself; add the remainder, if any, and if it be right, the sum will equal the given number.

Examples.

1. What is the square root of 529?

```
 . .
 529(23 root.
 4
 --
43)129
   129
   ---
```

3. What is the square root of 2? Ans. 1.4142+.

The decimals are found by annexing pairs of ciphers continually to the remainder for a new dividend. In this way, a surd root may be obtained to any assigned degree of exactness.

---

length of one side of this square, for that will be the number, which, multiplied into itself, will produce the given number of feet. Now by distinguishing 529 into periods, 5 2 9 we find the root or length of one side of the square will be expressed by two figures.

```
 . .
529(20
4
---
129

 . .
529(23
4
--
 129
 129
 ---
```

Now the greatest square number in 5, the left hand period, is 4, and its root 2; putting 2 in the quotient, I subtract 4 from the left hand period, and to 1, the remainder, bring down the next period, making the sum 129. Here it is plain that 2 is in the place of tens, because the root is to consist of two figures: its true value is therefore 20, and its square 400. Thence it appears that 400 feet of the boards are disposed of in a square form, measuring 20 feet on each side, and that there are 129 feet remaining to be added to the square, and in order that the form should continue square, it is necessary that the additions should be made upon two sides. Now the length of the two sides, to which the additions are to be made, is found by doubling 20=40, and dividing 129, the number of feet to be added, by 40, the length to which the addition is made, evidently gives the breadth of the addition. But if the length of the additions be only equal to the length of the sides to which they are made, there will be a deficiency at the corner of a small square, one side of which will be just equal to the width of the additions; the length upon which the addition is made, should therefore be increased by the breadth of the addition, and this is done by placing 3 on the right hand of the divisor. By this means, additions are made to the two sides 3 feet wide, and the corner filled up by a little square, measuring 3 feet on each side, which disposes of all the boards, and leaves them in the form of a complete square, 23 feet on each side.

3. What is the square root of 183.25? Ans. 13.5.

4. What is the square root of .0003272481? Ans. .01809.

5. What is the square root of 5499025? Ans. 2345.

6. What is the square root of $\frac{5}{12}$? Ans. .64549.

Reduce $\frac{5}{12}$ to a decimal, and then extract the root.

7. What is the square root of $\frac{25}{81}$? Ans. $\frac{5}{9}$*

8. What is the square root of $\frac{144}{729}$? Ans. $\frac{12}{27}$.

Application.

1. An army of 567009 men are drawn up in a solid body in the form of a square; what is the number of men in rank and file? Ans. 753.

2. What is the length of the side of a square which shall contain an acre, or 160 rods? Ans. 12.649+rods.

3. The area of a circle is 234.09 rods; what is the length of the side of a square of equal area? Ans. 15.3 rods.

4. The area of a triangle is 44944 feet; what is the length of the side of an equal square? Ans. 212 feet.

5. The diameter of a circle is 12 inches; what is the diameter of a circle 4 times as large? Ans. 24.

Circles are to one another as the squares of their diameters; therefore, square the given diameter, multiply or divide it by the given proportion, as the required diameter is to be greater or less than the given diameter, and the square root of the product or quotient will be the diameter required.

6. The diameter of a circle is 121 feet; what is the diameter of a circle one half as large? Ans. 85.5+ feet.

Having two sides of a right angled triangle given to find the other side.

RULE.†—Square the two given sides, and if they are the two sides which include the right angle, that is, the two shortest sides, add them together, and the square root of the sum will be the length of the longest side; if not, subtract the square of the less from that of the greater, and the square root of the remainder will be the length of the side required.

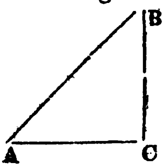
---

*When the terms of the fraction are complete powers, extract the root of the numerator for the numerator of the root, and the root of the denominator for the denominator of the root.

†A right angle is an angle that is formed by a line falling perpendicularly upon another line, as the angle C in the triangle A B C, and a right angled triangle is a triangle, which has one such angle. The rule is founded on the celebrated proposition of Pythagoras, which is the 47th proposition in the 1st Book of Euclid, viz: that *the square formed on the line subtending, or opposite to the right angle, in a right angled triangle, is equal to the sum of the squares formed on both the other sides*; that is, the square formed in the line A B is equal to the sum of the squares formed on the sides A C and C B, which may be demonstrated to be true in all cases.

Examples.

1. In the right angled triangle A B C, the side A C is 36 inches, and the side B C 27 inches; what is the length of the side A B?




```
 36    27
 36    27
 ---   ---
 216   189
108    54
----   ---
1296   729
```

```
1296 square of A C=36.
 729 square of B C=27.
----
2025 sum.

    2025(45 in. Ans.
    16
    --
 85)425
    425
    ---
```

2. Suppose a man travel east 40 miles, (from A to C) and then turn and travel north 30 miles; (from C to B) how far is he from the place (A) where he started? Ans. 50 miles.

3. A ladder 48 feet long will just reach from the opposite side of a ditch, known to be 35 feet wide, to the top of a fort; what is the height of the fort? Ans. 32.8+ feet.

4. A ladder 40 feet long, with the foot planted in the same place, will just reach a window on one side of the street 33 feet from the ground, and one on the other side of the street 21 feet from the ground; what is the width of the street? Ans. 56.66+ feet.

5. A line 81 feet long, will exactly reach from the top of a fort, on the opposite bank of a river, known to be 69 feet broad; the height of the wall is required? Ans. 42.425 feet.

6. Two ships sail from the same port, one goes due east 150 miles; the other due due north 252 miles; how far are they asunder? Ans. 293.25 miles.

To find a mean proportional between two numbers.

RULE.—Multiply the two given numbers together, and the square root of the product will be the mean proportional sought.

Examples.

1. What is the mean proportional between 4 and 36?

```
 36       144(12 Ans.
  4       1
 ---      --
 144    22)44
           44
           --
```

Then 4 : 12 : : 12 : 36

2. What is the mean proportional between 49 and 64? Ans. 56.

3. What is the mean proportional between 16 and 64? Ans. 32.

QUESTIONS.

1. What is Evolution?
2. What is meant by the root of any power?
3. How are roots denominated?
4. How is the square root denoted?
5. How are other roots denoted?
6. Is there any other way of denoting roots?
7. Has every number a root?
8. Can the complete root of all numbers be ascertained?
9. When is a power complete, and when incomplete?
10. What is the root of an incomplete power called?
11. How do you prepare any number or power for extracting its root?
12. How do you designate the periods?
13. What is shown by the number of periods?
14. How are decimals prepared for extracting their root?
15. What is the first step in the rule for extracting the square root?
16. What is the second?
17. What the third?
18. What the fourth?
19. What is the method of proof?
20. How do you extract the root of a Vulgar Fraction?
21. What is a square?
22. What proportion have circles to another?
23. When two sides of a right angled triangle are given, what is the rule for finding the other side?
24. How do you find a mean proportional between two numbers?

NOTES.

1. *Why do you subtract the square from the period in which it is taken?*
2. *Why do you double the root for a divisor?*
3. *In dividing, why is the right hand figure of the dividend excepted?*
4. *Why do you place the quotient figure in the divisor as well as in the root?*
5. *What is the 47th proposition in Euclid, which is referred to?*

TO EXTRACT THE CUBE ROOT.

The *cube root* of a number is a number which multiplied into its square, will produce that number. A *cube* is a solid body comprehended under six equal sides, each of which is an exact square, and its root is the length of one of the sides.

RULE.*—1. Having distinguished the given number into periods of three figures each, find the greatest cube in the left hand period, and place its root in the quotient.

2. Subtract the cube from the left hand period, and bring down the next period for a dividend.

3. Multiply the square of the quotient by 300, calling it the triple square, and the quotient by 30, calling it the triple quotient, and the sum of these call the divisor.

* The reason of the rule will appear by a consideration of the first example. Having distinguished the given number into periods, we find that the root will consist of two figures. Now if we suppose the given number 10648 to be so many solid feet of wood, which are to be piled into a cubical heap, the two figures of which the root is to consist will express the length of one side of that heap. By trial we find 8, whose root is 2, the greatest cube in the left hand period; we therefore place 2 for the first figure of the root, and subtract 8 from the left hand period. But as 2 is in the place of tens, its value is 20, and its

4. Seek how often the divisor may be had in the dividend, and place the result in the quotient.

5. Multiply the triple square by the last quotient figure, and write the product under the dividend; multiply the triple quotient by the square of the last quotient figure, and place this product under the last; under these write the cube of the last quotient figure, and call their sum the subtrahend.

---

cube 8 is 8000; therefore 8000 of the given number of feet are piled into a cubical heap, whose side is 20 feet, and there are 2648 feet to be added to the pile in such manner that it shall still retain its cubical form. In order to do this, it is evident that the additions must be made to 3 sides of the cube already formed. Here the rule directs to multiply the square of the last quotient figure by 300. This gives the superfices of the 3 sides to which the additions are to be made, as may be thus shown: 20 has been found to be the length of the several sides of the cube, $20\times20=400$, the superfices of one side; this multiplied by 3 gives $(400\times3=)$ 1200 for the superfices of the 3 sides, the same as by the rule for squaring 2, $(2\times2=4)$ and multiplying it by 300 $(4\times300=1200)$ is the same as squaring 20, and multiplying it by 3. Again, the rule directs to multiply the quotient figure by 30. Now it is evident that there will be three deficiencies between the additions which are made upon the 3 sides, of the length of those additions; that is, 3 deficiencies, each 20 feet long; or in the whole, $(20\times3=)$ 60 feet; but because the cipher is omitted in the quotient by the rule, and the 2 only used, we must annex the cipher to 3, the number of deficiencies, and multiply the 2 by 30 for the length of the deficiencies. These two, 1200 and $60=1260$, show the points upon the cube to which the additions are to be made. The 2648 feet being divided by this, shows the thickness of the additions to be made, which is 2 feet, therefore 2 is the other figure of the root. Now to see what timber is used in making these additions, we are directed first to multiply the triple square (1200, which is the superfices of the 3 sides to which the additions are made) by the last quotient figure. This gives $(1200\times2=)$ 2400 feet for the additions upon the 3 sides. Then to find how much it takes to fill up the deficiencies between the additions upon the sides, we are directed to multiply the triple quotient (60, the length of the deficiencies) by the square of the last quotient figure. This gives $(60\times4=)$ 240 feet, employed in filling the deficiencies between the other additions. The reason for multiplying the triple quotient by the square of the last quotient figure, is that two of the dimensions of this addition are just equal to the thickness of the additions upon the sides. But after these additions there is still evidently a deficiency at the corner, between the ends of the last additions, the 3 dimensions of which are just equal to the thickness of the other additions, and to fill this, we are therefore directed to cube the last quotient figure, $(2\times2\times2=8.)$ Then the quantities employed in these additions are 2400 feet, 240 feet, and 8 feet, which, added together, give 2648 feet, a sum just equal to the dividend, which shows that the cube is complete, measuring 22 feet on each side, and that all the 10648 feet of timber is used.

$$\begin{array}{l}
10648(20 \\
8 \\
\hline
2648
\end{array}$$

$$\begin{array}{rr}
2\times2\times300=1200 & 10648(22 \\
2\times30=\quad 60 & 8 \\
\hline
1260 &)2648 \\
\hline
 & 1200\times2=2400 \\
 & 60\times2\times2=\ 240 \\
 & 2\times2\times2=\quad 8 \\
\hline
 & 2648
\end{array}$$

The steps in this rule may be very clearly illustrated by the help of a cubical block, with other small blocks in the form of the several additions.

6. Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before; and so on till the whole is finished.

Examples.

1. What is the cube root of 10648?

2×2×300=1200 triple square.
2×30= 60 trip. quot.

```
       10648(22
  2×2×2=8
       ----
   1260)2648 dividend.
        ----
  1200×2=2400
  60×2×2= 240
  2×2×2=    8
        ----
        2648
        ----
        0000
```

Proof, 22×22×22=10648.

2. What is the cube root of 303464448? Ans. 672.

3. What is the cube root of 41.063625? Ans. 3.45

4. What is the cube root of 2? Ans. 1.25+

The decimals are obtained by annexing ciphers to the remainder, as in the square root, with this difference, that 3 instead of 2 are annexed each time.

5. What is the cube root of 27054036008? Ans. 3002.

6. What is the cube root of $\frac{1520}{5130}$? $\left(\frac{1520}{5130}\right)^{\frac{1}{3}}=\left(\frac{8}{27}\right)^{\frac{1}{3}}=\frac{2}{3}$ Ans.

7. What is the cube root of $\frac{2}{3}$? $\left(\frac{2}{3}\right)^{\frac{1}{3}}=\left(.666666+\right)^{\frac{1}{3}}=.873+$ Ans.

Application.

Solids of the same form are in proportion to one another as the cubes of their similar sides or diameters.

1. If a bullet weighing 72 lbs. be 8 inches in diameter, what is the diameter of a bullet weighing 9 lbs.?

```
8×8×8=512
lb.    lb.
72 : 512 : : 9
             9
    ----
72) 4608(64 the cube root of which
    432        is 4, the Answer.
    ----
     288
     288
```

2. A bullet 3 inches diameter weighs 4 lb. what is the weight of a bullet 6 inches diameter?

3×3×3=27 and 6×6×6=216
lb.
Thus 27 : 4 : : 216
Ans. 32 lbs.

3. If a ball of silver 12 inches in diameter be worth $600, what is the worth of another ball, the diameter of which is 15 inches? Ans. $1171.87+

4. If a cable 12 inches round require an anchor of 18 cwt. what must be the weight of an anchor for a 15 inch cable?

cwt. cwt. qr. lb.

$12^3 : 18 :: 15^3 : 35\ 0\ 17\frac{1}{2}$ Ans.

5. The diameter of a legal Winchester bushel is $18\frac{1}{2}$ inches, and its depth 8 inches; what must the diameter of that bushel be whose depth is $7\frac{1}{2}$ inches? Ans. 19.10671.

6. There is a cistern which contains 8204 solid inches, I demand the side of a cubical box which shall contain the same quantity. Ans. 14.12+ in.

7. A person wanted a cylindrick vessel of 3 feet deep, that shall hold twice as much as another of 23 inches deep, and 46 inches in diameter; what must be the diameter of the required vessel? Ans. 57.37 in.

Between two given numbers to find two mean proportionals.

RULE.—Divide the greater by the less, and extract the cube root of the quotient. Multiply the least given number by the root for the lesser, and this product by the same root for the greater of the two numbers sought.

Examples.

1. What are the two mean proportionals between 2 and 16?

$16 \div 2 = 8$ and $8^{\frac{1}{3}} = 2$

thus $2 \times 2 = 4$ the lesser,

and $4 \times 2 = 8$ the greater.

Proof. $2 : 4 :: 8 : 16$.

2. What are the two mean proportionals between 6 and 162? Ans. 18 and 54.

3. *TO EXTRACT THE ROOT OF ANY POWER.*

Rule 2.

1. Prepare the given number for extraction by pointing off from the place of units according the required root.

2. Find the first figure of the root by trial, subtract its power from the first period, and to the remainder bring down the first figure in the next period, and call these the *dividend.*

3. Involve the root already found to the next inferior power to that which is given, and multiply it by the number denoting the given power, for a *divisor.*

4. Find how many times the divisor may be had in the dividend, and the quotient will be another figure of the root.

5. Involve the whole root to the given power; subtract it from the given number as before, bring down the first figure of the next period to the remainder for a new dividend, to which find a new divisor, and so on till the whole is finished.

Examples.

1. What is the cube root of 48228544?

$$\begin{array}{l} \dot{}\dot{}\dot{} \\ 48228544(364 \\ 3^3=27 \\ \hline 3^2\times3=27)212 \text{ dividend.} \\ \hline 36^3=46656 \\ \hline 36^2\times3=3708)15725 \text{ 2d div'd.} \\ \hline 364^3=48228544 \end{array}$$

2. What is the fourth root of 19987173376? Ans. 376.

3. What is the sixth root of 191102976? Ans. 24.

4. What is the seventh root of 3404825447? Ans. 23.

5. What is the fifth root of 307682821106715625? Ans. 3145.

QUESTIONS.

1. What is the cube root of a number?
2. What is a cube?
3. What is its root?
4. What is the first step in the rule for extracting the root?
5. What is the second step?
6. What the third?
7. What the fourth?
8. What the fifth?
9. What the sixth?
10. When there is a remainder, how how do you proceed to find decimal places in the root?
11. What proportion have solids of the same form to one another?
12. How are two mean proportionals between two given numbers found?
13. What is the rule for extracting the roots of any powers?

NOTES.

1. *Why do you multiply the square of the quotient by 300?*
2. *Why the quotient by 30?*
3. *What is found by multiplying the triple square by the last quotient figure?*
4. *Why do you multiply the triple quotient by the square of the last quotient figure?*
5. *Why do you add to these the cube of the last quotient figure?*

SECTION II.

Arithmetical Progression.

A rank of numbers is in *Arithmetical Progression,* when they increase by common excess, or decrease by a common difference. When the numbers increase, they form an *ascending series,* and when they decrease, a *descending series.* Thus, 1, 2, 3, 4, &c. and 3, 6, 9, 12, &c. are ascending series, and 10, 9, 8, 7, &c. and 20, 16, 12, 8, &c. descending series.

The *terms* of the progression are the numbers which form the series. The first term and last term are called the *extreme.*

If any three of the five following things be given, the other two are readily found, viz. the first term, the last term, the number of terms, the common difference, and the sum of all the terms.

Problem I.

The first term, the last term, and the number of terms given, to find the sum of all the terms.

RULE.*—Multiply the sum of the extremes by the number of terms, and half the product will be the answer.

Examples.

1. The first term of an arithmetical progression is 1, the last term 21, and the number of terms 11; what is the sum of the series?

```
   21 last term.
    1 first term.
   --
   22
   11 number of terms.
   --
   22
  22
  ---
2)242
  ---
  121 Ans.
```

2. How many times does a common clock strike in 12 hours? Ans. 78 times.

3. Thirteen persons gave their donations to a poor man, in arithmetical progression, the first gave 2 cents, and the last 26; what did the poor man receive? Ans. $1.82.

* Suppose another series of the same kind with the given one, to be placed under it in an inverse order; then will the sum of every two corresponding terms be the same as that of the first and last; consequently, any one of these sums, multiplied by the number of terms, will give the whole sum of the two series, and half this sum will evidently be the sum of the given series; thus,

2	4	6	8	10	given series.
10	8	6	4	2	the same inverted.

$12+12+12+12+12=12\times5=60$ and $\frac{60}{2}=30=2+4+6+8+10$

Problem II.

The first term, the last term, and the number of terms given to find the common difference.

Rule.*—Divide the difference of the extremes by the number of terms, less 1, and the quotient will be the common difference.

Examples.

1. The extremes are 2 and 53, and the number of terms 18; what is the common difference?

18	53
1	2
—	—
17	17)51(3 Ans.
	51
	—

2. A man has 12 sons, whose ages are in arithmetical progression, the youngest is 2 years old, and the oldest 35; what is the common difference in their ages?

Ans. 3 years.

Problem III.

The first term, the last term, and common difference given, to find the number of terms.

Rule.†—Divide the difference of the extremes by the common difference, and the quotient, increased by 1, is the number of terms required.

Examples.

1. The extremes are 2 and 53, and the common difference 3; what is the number of terms?

53—2=51 and 3)51(17	17
3	1
—	—
21	18 Ans.
21	
—	

2. A man on a journey, travelled the first day 5 miles; the last day 35 miles, and increased his travel each day by 3 miles; how many days did he travel?

Ans. 11 days.

QUESTIONS.

1. When is a rank of numbers in Arithmetical Progression?
2. What is meant by an ascending series?
3. What by a descending?
4. What are the extremes?
5. What are the terms of a progression?
6. What is the first problem?
7. What the rule?
8. What the second? &c.

* The difference of the first and last terms evidently shows the increase of the first term, by all the subsequent additions, till it becomes equal to the last; and as the number of those additions is evidently one less than the number of terms, and the increase by every addition equal, it is plain that the total increase, divided by the number of additions, will give the difference at every one separately; whence the rule is manifest.

† By Problem II. the difference of the extremes, divided by the number of terms, less 1, gives the common difference; consequently, the same divided by the common difference, must give the number of terms less 1; hence this quotient, increased by 1, must be the answer to the question.

A Geometrical Progression.

A series of numbers is said to be in *Geometrical Progression*, when its terms increase by a constant multiplier, or decrease by a constant divisor. Thus, 2, 4, 8, 16, 32, &c. and 27, 9, 3, 1, are series in geometrical progression, the one increasing by a constant multiplication, by 2, and the other decreasing by a constant division, by 3.

The number by which the series is constantly increased or diminished, is called the *ratio*.

Problem I.

The first term, the last term, and the ratio given to find the sum of the series.

RULE.*—Multiply the last term by the ratio, and from the product subtract the first term, and the remainder divided by the ratio, less 1, will give the sum of the series.

Examples.

1. The first term of a series in geometrical progression is 1, the last term is 243, and the ratio 3; what is the sum of the series?

$$\begin{array}{r} 243 \\ 3 \\ \hline 729 \\ 1 \\ 3-1=2)728 \\ \hline 364 \text{ Ans.} \end{array}$$

2. The extremes of a geometrical progression are 1 and 65536, and the ratio 4; what is the sum of the series? Ans. 87381.

3. The extremes are 1024 and 59049, and the ratio $1\frac{1}{2}$; what is the sum of the series?
Ans. 175099.

---

* The reason of the rule may be shown thus: take any series, as 1, 3, 9, 27, 81, 243, &c. multiply it by the ratio, and it will produce the series, 3, 9, 27, 81, 243, 729, &c. Let the given series be what it will, it is plain that the sum of the second series will be as many times that of the first as is expressed by the ratio. Now subtract the first series from the second, and it gives 729—1, which is evidently as many times the sum of the first series as is expressed by the ratio less 1; consequently, $\frac{729-1}{3-1}=$ the sum of the proposed series, and is the rule; or 729 is the last term multiplied by the ratio, 1 is the first term, and 3—1 is the ratio less 1, and the same will hold, whatever be the series.

When a geometrical series consists of an even number of terms, the product of the extremes is equal to the product of any two means equally distant from the extremes; and when the number of terms is odd, the product of the extremes is equal to the square of the middle term, or to the product of any two means equally distant from them.

Problem II.

The first term and ratio given to find any other term assigned.

RULE.*—1. Write a few of the leading terms of the series, and place their indices over them, beginning with a cipher, and add together the most convenient indices to make an index less by 1 than the number, expressing the place of the term sought.

2. Multiply the terms of the series belonging to those indices together for a dividend, and raise the first term to a power whose index is 1 less than the number of terms multiplied for a divisor; divide the dividend by the divisor, and the quotient will be the term sought.

NOTE.—When the first term of the series is equal to the ratio, the indices must begin with a unit, and the indices added must make the entire index of the term required; and the product of the several terms found as above, will be the term required.

Examples.

1. The first term of a geometrical series is 2, the number of terms 13, and the ratio 2; what is the last term?

$\overset{1}{2}\ \overset{2}{4}\ \overset{3}{8}\ \overset{4}{16}\ \overset{5}{32}$ indices. leading terms.

Then $3+5+5=$ind. to 13th ter.

And $8\times32\times32=8192$ Ans.

In this example the indices begin with 1, because the first term and ratio are the same, and such of the indices are chosen as will make up the entire index of the term required.

2. If the first term of a series be 5, and the ratio 3, what is the 11th term?

$\overset{0}{5}\ .\ \overset{1}{15}\ \overset{2}{45}\ \overset{3}{135}\ \overset{4}{405}$ indices. leading terms.

Then $2+4+4$

$\frac{45\times405\times405}{5^2}=\frac{7381125}{25}=295245$ Ans.

The number of terms is 3, and $3-1=2$ is the power to which 5, the first term, is to be raised for the divisor.

3. What debt will be discharged in 12 months by paying \$1 the first month, \$2 the second, \$4 the third, and so on, each succeeding payment being double the last; and what will be the last payment?

Ans. { \$4095 the debt. \$2048 last pay't.

4. A gentleman being asked to dispose of a horse, said he would sell him on condition of having 1 cent for the first nail in his shoes, 2 cents for the second, 4 cents for the third, and so on, doubling the price of every nail to 32, the number of nails in his four shoes; what was the price of the horse at that rate?

Ans. \$42949672.95.

---

* When the first term is equal to the ratio, the reason of the rule is obvious; for as every term is some power of the ratio, and the indices point out the number of factors, it is evident from the nature of multiplication, that the product of any two terms will be another term corresponding with the index, which is the sum of the indices standing over those respective terms. And when the series does not begin with the ratio, it will be seen that every term after the two first, contains some power of the ratio, multiplied by the first term, and therefore the rule is in this case equally evident.

QUESTIONS.

1. When is a series of numbers in Geometrical Progression?
2. What is meant by the ratio?
3. What is the first problem?
4. What is the rule?
5. What is the second problem?
6. What is the rule?

E. Annuities.

An *Annuity* is a sum of money payable every year for a certain number of years, or forever.

When the debtor keeps the annuity in his own hands beyond the time of payment, it is said to be in *arrears*.

The sum of all the annuities for the time they have been forborn, together with the interest due upon each, is called the *amount*.

If an annuity be to be bought off, or paid all at once at the beginning of the first year, the price which ought to be given for it, is called the *present worth*.

Case I.

To find the amount of an annuity at simple interest.

RULE.*—Find the sum of the natural series of numbers, 1, 2, 3, &c. to the number of years less 1; multiply this sum by one year's interest of the annuity, and the product will be the whole interest due upon the annuity; to this product add the product of the annuity and time, and the sum will be the amount required.

Examples.

1. What is the amount of an annuity of $60 for 5 years, at 6 per cent simple interest?

1+2+3+4=10
$3.60= 1 year's int. of $60
10
———
36.00
300.=$60×5
———
336.00 am't required.

2. What will an annuity of $90 amount to in 6 years at 6 per cent simple interest?
Ans. $621.

3. If a salary of $750 be forborn 4 years, what will it amount to at the end of that time, allowing 4 per cent, simple interest?
Ans. $3180.

---

* Whatever the time is, there is due upon the first year's annuity, as many years' interest as the whole number of years less 1; and gradually one less upon every succeeding year to the last but one, upon which there is due only one year's interest, and none upon the last: therefore, in the whole, there is due as many year's interest of the annuity, as the sum of the series, 1, 2, 3, 4, &c. to the number of years less 1. Consequently, one year's interest, multiplied by this sum, must be the whole interest due; to which, if all the annuities be added, the sum is plainly the amount.

Case II.

To find the present worth of an annuity at simple interest.

Rule.*—Find the present worth of each year by itself, discounting from the time it becomes due, and the sum of all these will be the present worth required.

Examples.

1. What is the present worth of $400 per annum, to continue 4 years at 6 per cent, simple interest?

106 : 100 :: 400 : 377.358 = p.w. 1y.
112 : 100 :: 400 : 357.142 = " 2d
118 : 100 :: 400 : 338.983 = " 3d
124 : 100 :: 400 : 322.580 = " 4th

$1396.063 pr't w'th.

2. What is the present worth of an annuity of £100 to continue 5 years at 6 per cent, simple interest? Ans. £425.18s 9½d.

3. What is the present worth of a pension of $500 to continue 4 years, at 5 per cent, simple interest? Ans. $1782.183.

QUESTIONS.

1. What is an annuity?
2. When is it said to be in arrears?
3. What is meant by the present worth?
4. What is the amount?
5. How is the amount of an annuity found at simple interest?
6. How is the present worth found?

I. Position.

Position is the rule which discovers the true number by the use of false, or supposed numbers. It is of two kinds, *single* and *double*.

SINGLE POSITION.

Single Position teaches to resolve those questions whose results are proportional to their suppositions.

Rule.—Take any number and perform the same operations with it as are described to be performed in the question: Then say, as the result of the operation is to the given number, so is the supposed number to the true one required.

* The reason of this rule is manifest from the nature of discount.

Examples.

1. A's age is double that of B, and B's age is triple that of C, and the sum of all their ages is 140; what is the age of each?

Suppose A's age to be 48
Then will B's $=\frac{48}{2}=24$
And C's $=\frac{24}{3}=8$

80 sum.

Then 80 : 140 : : 48 : 84 A's age.
24 : 42 B's age.
8 : 14 C's age.

140 proof.

2. What number is that which being increased by $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ itself, the sum will be 125? Ans. 60.

3. What number is that whose 6th part exceeds its 8th part by 20? Ans. 480.

4. A vessel has 3 cocks, the first will fill it in 1 hour, the second in 2, the third in 3; in what time will they all fill it together? Ans. $\frac{6}{11}$ hour.

5. A person, after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money, had \$60 left; what had he at first? Ans. \$144.

6. What number is that, from which, if 5 be subtracted, $\frac{2}{3}$ of the remainder will be 40? Ans. 65.

7. A gentleman had a certain number of dollars in his purse; the sum of the third, fourth, and sixth part of them made 54; how many were there in the purse? Ans. \$72.

DOUBLE POSITION.

Double Position teaches to resolve questions by making two suppositions of false numbers.

Rule.*—1. Take any two numbers and proceed with each according to the condition of the question, noting the errors.

2. Multiply the first supposed number by the last error, and the last supposed number by the first error; and, if the errors be *alike*, (that is, both too great, or both too small,) divide the difference of the products by the difference of the errors; but if *unlike*, divide the sum of the products by the sum of the errors, and the quotient will be the answer.

Examples.

1. There is a fish, whose head is 9 feet long, his tail is as long as his head and half the length of his body, and his body is as long as his head and tail; what is the whole length of the fish?

---

* This rule is founded on the supposition that the first error is to the second, as the difference between the true and first supposed is to the difference between the true and second supposed number; when that is not the case, the exact answer to the question cannot be found by this rule.

Suppose	40 ft.	Again suppose	60 ft.
His body	20	body	30
His tail	19	tail	24
His head	9	head	9
Sum	48	sum	63
First error	8 —	second error—	3

40	8
X	
60	3
8	40
480	120
120	

8—3=5)360(72 feet, Ans.
35

10

2. A gentleman has 2 horses and a saddle worth $50; if the saddle be put on the first horse, his value will be double that of the second; but if it be put on the second, his value will be triple that of the first; what is the value of each horse?
Ans. 1st horse $30, 2d $40.

3. A man having driven his swine to market, viz. hogs, sows and pigs, received for them all £50, being paid for each hog 18s. for each sow 16s. and for each pig 2s.; there were as many hogs as sows, and for every sow 3 pigs; what was the number of each sort?
Ans. 25 hogs, 25 sows, 75 pigs.

4. A and B lay out equal shares in trade; A gains $126, and B loses $87, then A's money is double that of B; what did each lay out? Ans. $300.

5. A and B have both the same income; A saves ¼ of his yearly, but B, by spending $50 per annum more than A, at the end of 4 years, finds himself $100 in debt; what is their income, and what do they spend per annum?
Ans. $125 their inc. per ann.
A spends $100 } per ann.
B spends $150 }

QUESTIONS.

1. What is Position?
2. Of how many kinds is it?
3. What does Single Position teach?
4. What is the rule?
5. What does Double Position teach?
6. What is the rule?

Permutation of Quantities.

Permutation of Quantities is a rule which shows how many different ways the order or position of any given number of things may be varied.

Problem I.

To find the number of permutations, or changes, that can be made of any given number of things, all different from each other.

RULE.*—Multiply all the terms of the natural series of numbers from 1 up to the given number, continually together, and the last product will be the answer required.

Examples.

1. How many changes can be made of the letters in the word *and?*

1	a n d	Proof.
2	a d n	
—	n a d	
2	n d a	
3	d a n	
—	d n a	

6 Ans. or $1 \times 2 \times 3 = 6$ Ans.

2. How many days can 7 persons be placed in a different position at dinner? Ans. 5040 days.

3. How many changes may be rung on 6 bells? Ans. 720.

4. How many changes can be made in the position of the 8 notes of musick? Ans. 40320.

5. How many changes may be rung on 12 bells, and how long would they be in ringing, supposing 10 changes to be rung in one minute, and the year to consist of 365 days, 5 hours and 49 min'ts? Ans. 479001600 changes, and 91 years, 26d. 22h. 41m. time.

Problem II.

To find how many changes can be made out of a given number of different things, by taking any given number at a time.

RULE.—Take a series of numbers, beginning at the number of things given, and decreasing by 1 till the number of terms taken be equal to the number of things to be taken at a time, and the product of all these terms will be the answer.

Examples.

1. How many changes can be rung on 3 bells out of 8?
$8 \times 7 \times 6 = 336$ Ans.

2. How many words can be made with 5 letters of the alphabet, supposing 24 letters in all, and that a number of consonants alone will make a word?
Ans. 5100480.

QUESTIONS.

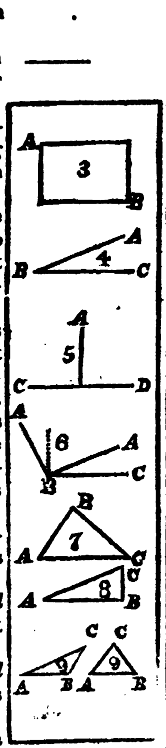
1. What is Permutation of Quantities?
2. How do you find the number of changes that can be made of any number of things all different from each other?

* The reason of the rule may be shown thus: any thing, *a*, is capable of only one position, as *a*. Any two things, *a* and *b*, are capable of only two variations, as *ab*, *ba*; when a number is expressed by 1×2. If there be three things, *a*, *b*, and *c*, then any two of these, leaving out the third, will have 1×2 variations; and consequently, when the third is taken in, these will be $1 \times 2 \times 3$ variations; and so on as far as you please.

SECTION III.

I. Mensuration of Superficies.

Definitions.



1. A *point* is position without magnitude.

2. A *line* is length without breadth or thickness.

3. A *superficies*, or *surface*, is a figure having length and breadth without thickness.

4. An *angle* is the opening between two lines, having different directions, and meeting in a point.

5. A *right angle* is formed by one right line falling perpendicularly upon another.

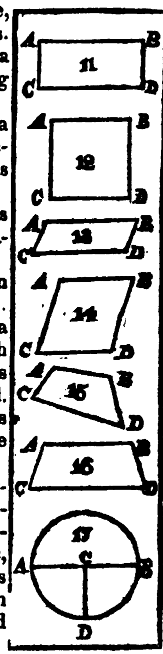
6. An *oblique angle* is formed by two oblique lines, and may be either greater or less than a right angle.

7. A *triangle* is a figure having 3 sides and 3 angles.

8. A *right angled triangle* has one right angle.

9. An *oblique angled triangle* has all its angles oblique.

10. A *parallelogram* is a four sided figure which has both pair of its opposite sides parallel, of which there are 4 varieties, viz. the rectangle, square, rhomboid & rhombus.



11. A *rectangle* is a parellelogram having all its angles right.

12. A *square* is a figure having four equal sides, and all its angles right.

13. A *rhomboid* is an oblique angled parallelogram.

14. A *rhombus* is an equal sided rhomboid.

15. A *trapezium* is a 4 sided figure which has neither pair of its opposite sides parallel.

16. A *trapezoid* has one pair of its opposite sides parallel.

17. A *circle* is a figure bounded by a continued curve line, called the *circumference*, every part of which is equally distant from a point within called the *centre*.

18. The *radius* of a circle is a right line from the centre to the circumference.

19. The *diameter* of a circle is a right line extending thro' the centre, & terminated by the circumference.

The area of any figure is the space contained within the bounds of its surface, without any regard to thickness, and is estimated by the number of squares contained in the same; the side of those squares being either an inch, a foot, a yard, a rod, &c. Hence the area is said to be so many square inches, square feet, square yards, or square rods, &c.

Problem I.

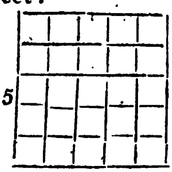
To find the area of a parallelogram, whether it be a square, a rectangle, a rhombus, or a rhomboid.

RULE.—Multiply the length by the breadth, or perpendicular height, and the product will be the area.

Examples.

1. What is the area of a square whose side is 5 feet?

5
5
—
Ans. 25 feet.

5

5

2. What is the area of a rectangle whose length is 9, and breadth 4 feet? Ans. 36 feet.

3. What is the area of a rhombus whose length is 12 rods, and perpendicular height 4?
Ans. 48 rods.

4. What is the area of a rhomboid 24 inches long, and 8 wide?
Ans. 192 inches.

5. How many acres in a rectangular piece of ground 56 rods long, and 26 wide?
$56 \times 26 \div 160 = 9\frac{1}{10}$ acres, Ans.

Problem II.

To find the area of a triangle.

RULE 1.—Multiply the base by half the perpendicular height, and the product will be the area.

RULE 2.—If the three sides only are given, add these together, and take half the sum; from the half sum subtract each side separately; multiply the half sum and the three remainders continually together, and the square root of the last product will be the area of the triangle.

Examples.

1. How many square feet in a triangle whose base is 40 feet, and height 30 feet?

40 base.
15 = ½ perpendicular height.
——
200
40
——
600 feet. Ans.

2. The base of a triangle is 6.25 chains, and its height 5.20 chains; what is its area?
Ans. 16.25 sq. chains.

3. What is the area of a triangle whose three sides are 13, 14 and 15 feet?

$13 + 14 + 15 = 42$
and $42 \div 2 = 21 = \frac{1}{2}$ sum.
21 21 21
13 14 15 and $21 \times 6 \times 7 \times 8 = 7056$
——
Rem. 8 7 6

Then $\overline{7056}|^{\frac{1}{2}} = 84$ feet, Ans.

4. The three sides of a triangle are 16, 11 and 10 rods; what is the area? Ans. 54.299 rods.

Problem III.

To find the area of a trapezoid.

RULE.—Multiply half the sum of the two parallel sides by the perpendicular distance between them, and the product will be the area.

Examples.

1. One of the two parallel sides of a trapezoid is 7.5 chains and the other 12.25, and the perpendicular distance between them is 15.4 chains; what is the area?

```
  12.25
   7.5
 ------
2)19.75
 ------
   9.875
   15.4
 -------
  39500
 49375
 9875
 --------
 152.0750 sq. chains, Ans.
```

2. How many square feet in a plank 12 feet 6 inches long, and at one end, 1 foot 3 inches, and at the other 11 inches wide? Ans. $13\frac{13}{24}$ feet.

3. What is the area of a piece of land 30 rods long and 20 rods wide, at one end, and 18 rods at the other? Ans. 570 rods.

4. What is the area of a hall 32 feet long, and 22 feet wide at one end, and 20 at the other? Ans. 672 feet.

Problem IV.

To find the area of a trapezium, or an irregular polygon.

RULE.—Divide it into triangles, and then find the area of these triangles by Problem II. and add them together.

Examples.

1. A trapezium is divided into two triangles, by a diagonal 42 rods long, and the perpendiculars let fall from the opposite angles of the two triangles, are 18 rods and 16 rods; what is the area of the trapezium?

```
 42     42     336
  9      8     378
---    ---     ---
378    336     704 rods, Ans.
```

2. What is the area of a trapezium whose diagonal is $108\frac{1}{2}$ feet, and the perpendiculars $56\frac{1}{4}$ and $60\frac{3}{4}$ feet? Ans. $6347\frac{1}{4}$ feet.

3. How many square yards in a trapezium whose diagonal is 65 feet, and the perpendiculars let fall upon it 28 and 33.5 feet? Ans. $222\frac{1}{12}$ yds.

Problem V.

To find the diameter and circumference of a circle, either from the other.

Rule 1. As 7 is to 22, so is the diameter to the circumference, and as 22 is to 7, so is the circumference to the diameter.

Rule 2. As 113 is to 355, so is the diameter to the circumference, and as 355 is to 113, so is the circumference to the diameter.

Rule 3. As 1 is to 3.1416, so is the diameter to the circumference, and as 3.1416 is to 1, so is the circumference to the diameter.

Examples.

1. What is the circumference of a circle whose diameter is 14ft.

By Rule 1.

As 7 : 22 :: 14 : 44 Ans.

By Rule 2.

As 113 : 355 :: 14 : $43\frac{111}{113}$ Ans.

By Rule 3.

As 1 : 3.1416 :: 14 : 43.9824 Ans.*

2. Supposing the diameter of the earth to be 7958 miles, what is its circumference?

Ans. 25000.8528 miles.

3. What is the diameter of a circle whose circumference is 50 rods? By Rule 1.

As 22 : 7 :: 50 : 15.9090 Ans.

By Rule 2.

As 355 : 113 :: 50 : 15.9155 Ans.

By Rule 3.

As 3.1416 : 1 :: 50 : 15.9156 Ans.

4. Supposing the circumference of the earth to be 25000 miles, what is its diameter?

Ans. $7957\frac{3}{4}$ nearly.

Problem VI.

To find the area of a circle.†

Rule.—Multiply half the circumference by half the diameter, and the product will be the area.

---

* These three methods do not exactly agree, but the last is the most correct. The exact proportion between the diameter and circumference of a circle has not yet been ascertained.

† The following are some of the most useful problems relating to the circle.

1. Circumference × diameter, $\frac{1}{4}$ the product=the area.
2. Square of diameter × .7854=area.
3. Square of circumference × .07958=area.
4. As 14 : 11 :: square of diameter : area.
5. As 88 : 7 :: square of circumference : area.
6. Diameter × .8862=side of an equal square.
7. Circumference × .2821=side of an equal square.
8. Diameter × .7071=side of an inscribed square.
9. Circumference × .2251=side of an inscribed square.
10. Area × .6366=side of an inscribed square.
11. Side of a square × 1.128=diameter of an equal circle.
12. Side of a square × 3.545=circumference of an equal circle.

Examples.

1. What is the area of a circle whose diameter is 7 and circumference 22 feet?

11 = ½ circumference.
3.5 = ½ diameter.

55
33

38.5 feet Ans.

2. What is the area of a circle whose diameter is 1, and circumference 3.1416? Ans. .7854

3. What is the area of a circle whose diameter is 10 rods, and circumference 31.416? Ans. 78.54 rods.

4. How many square chains in a circular field, whose circumference is 44 chains, and diameter 14? Ans. 154 chains.

5. How many square feet in a circle whose circumference is 63 feet? Ans. 315 feet.

Problem VII.

The area of a circle given to find the diameter and circumference.

Rule.—1. Divide the area by .7854, and the square root of the quotient will be the diameter.

2. Divide the area by .07958, and the square root of the quotient will be the circumference.

Examples.

1. What is the diameter of a circle whose area is 154 rods?

```
.7854)154.0000(196(14 rods, Ans.
      7854     1
      -----   ----
      75460  24)96
      70686     96
      -----     --
       47740
       47124
       -----
         616
```

2. The area of a circle is 78.5 feet; what is its circumference? Ans. 31.4 feet.

3. I demand the length of a rope to be tied to a horse's neck, that he may graze upon 7854 square feet of new feed every day for 4 days, one end of the rope being each day fastened to the same stake?

1st circle contains 7854 feet ÷ .7854 = 10000, and √10000 = 100 diam. ÷ 2 = 50 feet, the 1st rope. 2d circle contains 15708 ÷ 7854 = 20000, and √20000 = 141½ 70½ feet, second rope, &c.

1st rope	50	feet.	Ans.
2	—	70½ feet.	
3	—	86½ feet	
4	—	100 feet.	

Problem VIII.

To find the area of an oval, or ellipsis.

Rule.—Multiply the longest and shortest diameters together, and the product by .7854; the last product will be the area.*

* The longest diameter of an ellipsis is called the *transverse*, and the shortest the *conjugate* diameter.

Examples.

1. What is the area of an oval whose longest diameter is 5 feet, and shortest 4 feet?
$5 \times 4 \times .7854 = 15.708$ ft. Ans.

2. What is the area of an oval whose longest diameter is 21, and shortest 17?
Ans. 280.3878.

Problem IX.

To find the area of a globe or sphere.

RULE.—Multiply the circumference by the diameter, and the product will be the area.

Examples.

1. How many square feet in the surface of a globe whose diameter is 14 inches and circumference 44?
$44 \times 14 = 616$ Ans.

2. How many square miles in the earth's surface, its circumference being 25000, and its diameter 7957¾ miles?
Ans. 198943750.

3. What is the area of the surface of a cannon shot, whose diameter is one inch?
Ans. 3.1416 inches.

4. How many square inches in the surface of an 18 inch artificial globe? Ans. 1017.8784.

2. Mensuration of Solids.

Definitions.

1. A *solid* is a figure having three dimensions, viz. length, breadth and thickness.

2. A *prism* is a body whose ends are any equal and similar plane figures, and whose sides are parallelograms.

3. A *cube* is a body having six equal sides, all of which are squares.

4. A *parallelopipedon* is a body having six rectangular sides, every opposite pair of which are equal and parallel.

5. A *cylinder* is a round prism, having circles for its ends.

6. A *pyramid* is a solid whose base is any plane figure, and whose sides are triangular, meeting in a point at the top called a vertex.

7. A *cone* is a round pyramid, having a circle for its base.

8. A *sphere* is a solid bounded by one continued convex surface, every part of which is equally distant from a point within called the centre.

Mensuration of Solids teaches to determine the spaces included by contiguous surfaces, and the sum of the measures of these including surfaces is the whole surface of the body. The *measure* of a solid is called its solidity, capacity, or content. The content is estimated by the number of cubes, whose sides are inches, or feet, or yards, &c. contained in the body.

Problem I.

To find the solidity of a cube.

RULE—Cube one of its sides, that is, multiply the side by itself, and that product by the side again, and the last product will be the answer.

Examples.

1. If the length of the side of a cube be 22 feet, what is its solidity?
$22 \times 22 \times 22 = 10648$ Ans.

2. How many cubick inches in a cube whose side is 24 inches? Ans. 13824.

Problem II.

To find the solidity of a parallelopipedon.

RULE.—Multiply the length by the breadth, and that product by the depth, the last product will be the answer.

Examples.

1. What is the content of a parallelopipedon whose length is 6 feet, its breadth $2\frac{1}{2}$ feet, and its depth $1\frac{3}{4}$ feet?
$6 \times 2.5 \times 1.75 = 26.25$ or $26\frac{1}{4}$ feet.

2. How many feet in a stick of hewn timber 30 feet long, 9 inches broad, and 6 inches thick? Ans. $11\frac{1}{4}$ feet.

Problem III.

To find the side of the largest square stick of timber that can be hewn from a round log.

RULE.—Extract the square root of twice the square of the semidiameter at the smallest end for the side of the stick when squared.

Examples.

1. The diameter of a round log at its smallest end is 16 inches, what will be the side of the largest squared stick of timber that can be hewn from it?
$\sqrt{8 \times 8 \times 2} = 11.31$ in. Ans.

2. The diameter at the smallest end being 24 inches, how large square will the stick of timber hew? Ans. 16.97 in.

Problem IV.

To find the solidity of a prism, or cylinder.

RULE.—Multiply the area of the end by the length of the prism, for the content.

Examples.

1. What is the content of a triangular prism, the area of whose end is 2.7 feet, and whose length is 12 feet?
$2.7 \times 12 = 32.4$ ft. Ans.

2. What number of cubick feet in a round stick of timber, whose diameter is 18 inches, and length 20 feet? Ans. 35.343.

Problem V.

To find the solidity of a pyramid or cone.

RULE.—Multiply the area of the base by the height, and one third of the product will be the content.

Examples.

1. What is the content of a cone whose height is $12\frac{1}{2}$ feet, and the diameter of the base $2\frac{1}{2}$ feet?
$2\frac{1}{2}\times2\frac{1}{2}=\frac{5}{2}\times\frac{5}{2}=\frac{25}{4}=6\frac{1}{4}$ and
$6\frac{1}{4}\times.7854\times12\frac{1}{2}\div3=20.453125.$

2. What is the content of a triangular pyramid, its height being $14\frac{1}{2}$ feet, and the sides of its base being 5, 6, and 7 feet?
Ans. 71.035+

Problem VI.

*To find the solidity of a sphere.**

RULE.—Multiply the cube of the diameter by .5236, or multiply the square of the diameter by $\frac{1}{6}$ the circumference.

Examples.

1. What is the content of a sphere whose diameter is 12 inches? $12\times12\times12\times.5236=$ 904.7808 Ans.

2. What is the solid content of the earth, its circumference being 25000 miles?
Ans. 263858149120 miles.

3. Gauging.

Gauging teaches to measure all kinds of vessels, as pipes, hogsheads, barrels, &c.

RULE.—To the square of the bung diameter add the square of the head diameter; multiply the sum by the length, and the product by .0014 for ale gallons, or by .0017 for wine gallons.

Examples.

1. What is the content of a cask whose length is 40 inches, and its diameters 24 and 32 inches?
$\overline{32\times32}+\overline{24\times24}\times40=64000$ A.
$\left.\begin{array}{l}64000\times.0014=89.6 \text{ a. gal.}\\ 64000\times.0017=108.8 \text{ w. gal.}\end{array}\right\}$ A.

2. What is the content of a cask whose length is 20 inches, and diameters 12 and 16?
Ans. $\left\{\begin{array}{l}11.2 \text{ a. gal.}\\ 13.6 \text{ w. gal.}\end{array}\right.$

* The surface of a sphere is found by multiplying its diameter by its circumference.

SECTION IV.

PHILOSOPHICAL MATTERS.

I. Of the fall of heavy Bodies.

Heavy bodies, near the surface of the earth, fall one foot the first quarter of a second, three feet the second quarter, five feet the third quarter, and seven feet the fourth quarter, equal to 16 feet in the first second. The velocities acquired by falling bodies, are in proportion to the squares of the times in which they fall; that is, if 3 bullets be dropped at the same time, and the first be stopped at the end of the first second, the second at the end of the second, and the third at the end of the third, the first will have fallen 16 feet, the second, ($2\times2=4$) four times 16, equal to 64; and the third, ($3\times3=9$) nine times 16, equal to 144 feet, and so on. Or if 16 feet be multiplied by so many of the odd numbers beginning at 1, as there are seconds in the given time, these several products will be the spaces passed through in each of the several seconds, and their sum will be the whole distance fallen.

1. *The velocity given, to find the space fallen through.*

RULE.—Divide the velocity in feet by 8, and the square of the quotient will be the space fallen through to acquire that velocity.

1. From what height must a body fall to acquire the velocity of a cannon ball, which is about 660 feet per second?
$660\div8=82.5$ and $82.5\times82.5=6806.25$ft. $=1\frac{37}{128}$ miles, Ans.

2. From what height must a body fall to acquire a velocity of 1200 feet per second?
Ans. 22500 feet.

2. *The time given to find the space fallen through.*

RULE.—Multiply the time in seconds by 4, and the square of the product will be the space fallen through in the given time.

1. How many feet will a body fall in 5 seconds? $5\times4=20$, and $20\times20=400$ feet, Ans.

2. A stone, dropped into a well, reached the bottom in 3 seconds; what was its depth? $3\times4=12$, and $12\times12=144$ feet, Ans.

3. Ascending bodies are retarded in the same ratio that descending bodies are accelerated; therefore, if a ball, fired upwards return to the earth in 16 seconds, how high did it ascend? The ball being half the time, or 8 seconds, its ascent; therefore, $8\times4=32$, and $32\times32=1024$ ft. Ans.

3. *The velocity per second given to find the time.*

RULE.—Divide the given velocity by 8, and one fourth part of the quotient will be the answer.

1. How long must a body be falling to acquire a velocity of 160 feet per second? $160 \div 8 = 20$, and $20 \div 4 = 5$ seconds, Ans.

2. How long must a body be falling to acquire a velocity of 400 feet per second? Ans. $12\frac{1}{2}$ seconds.

4. *The space given to find the time the body has been falling.*

RULE.—Divide the square root of the space fallen through by 4, and the quotient will be the time.

1. In how many seconds will a body fall 400 feet? $\sqrt{400} = 20$, and $20 \div 4 = 5$ seconds, Ans.

2. In how many seconds will a bullet fall through a space of 11025 feet? Ans. $26\frac{1}{4}$ seconds.

5. *To find the velocity per second with which a body will begin to descend at any distance from the earth's surface.*

RULE.—As the square of the earth's semi-diameter is to 16 feet, so is the square of any other distance from the earth's centre, inversely, to the velocity with which it begins to descend per second.

1. Admitting the semi-diameter of the earth to be 4000 miles, with what velocity per second will a body begin to descend, if raised 4000 miles above the earth's surface? As $4000 \times 4000 : 16 :: 8000 \times 8000 : 4$ feet, Ans.

2. How high above the earth's surface must a ball be raised to begin to descend with a velocity of 4 feet per second? Ans. 4000 miles.

6. *To find the velocity acquired by a falling body, per second, at the end of any given period of time.*

RULE.—Multiply the perpendicular space fallen through by 64, and the square root of the product is the velocity required.

1. What velocity per second does a ball acquire by falling 225 feet? $225 \times 64 = 14400$, and $\sqrt{14400} = 120$, Ans.

2. If a ball fall 484 feet in $5\frac{1}{2}$ seconds, with what velocity will it strike? Ans. 176.

7. *The velocity with which a body strikes, given to find the space fallen through.*

RULE.—Divide the square of the velocity by 64, and the quotient will be the space required.

1. If a ball strike the ground with a velocity of 56 feet per second, from what height did it fall? $56 \times 56 \div 64 = 49$ feet, Ans.

2. If a stream move with a velocity of 12.649 feet per second, what is its perpendicular fall? Ans. $2\frac{1}{2}$ feet.

8. *To find the force with which a falling body will strike.*

RULE.—Multiply its weight by its velocity, and the product will be the force.

1. If a rammer for driving piles, weighing 4500 pounds, fall through the space of 10 feet, with what force will it strike?
$\sqrt{10\times64}=25.3=$ velocity, and $25.3\times4500=113850$ lb. Ans.

2. With what force will a 42lb. cannon ball strike, dropped from a height of 225 feet?
Ans. 5040 lb.

2. Of Pendulums.

The time of a vibration, in a cycloid, is to the time of a heavy body's descent through half its length as the circumference of a circle to its diameter; therefore to find the length of a pendulum vibrating seconds, since a falling body descends 193.5 inches in the first second, say, as $3.1416\times3.1416 : 1\times1 :: 193.5 : 19.6$ inches $=\frac{1}{2}$ the length of the pendulum, and $19.6\times2=39.2$ inches, the length.

1. *To find the length of a pendulum that will swing any given time.*

RULE.—Multiply the square of the time in seconds by 39.2, and the product will be the length required in inches.

1. What are the lengths of three pendulums, which will swing respectively, $\frac{1}{2}$ seconds, seconds and 2 seconds?

$$\left.\begin{array}{l} .5\times.5\times39.2=9.8\text{in. for } \tfrac{1}{2} \text{ seconds.} \\ 1\times1\times39.2=39.2\text{in. for seconds.} \\ 2\times2\times39.2=156.8\text{in. for 2 seconds.} \end{array}\right\}\text{Ans.}$$

2. What is the length of a pendulum, which vibrates 4 times in a second? $.25\times.25\times39.2=2.42$ inches, Ans.

3. Required the lengths of 2 pendulums, which will respectively swing minutes and hours?

$$\left.\begin{array}{r} 60\times60\times39.2=141120\text{in.}=2\text{m. } 1200 \text{ feet.} \\ 3600\times3600\times39.2=508032000=8018\text{m. } 960 \text{ feet.} \end{array}\right\}\text{Ans.}$$

2. *To find the time which a pendulum of a given length will swing.*

RULE.—Divide the given length by 39.2, and the square root of the quotient will be the time in seconds.

1. In what time will a pendulum 9.8 inches in length, vibrate?
$\sqrt{9.8\div39.2}=.5$, or $\frac{1}{2}$ second, Ans.

2. I observed that while a ball was falling from the top of a steeple, a pendulum 2.45 inches long, made 10 vibrations; what was the height of the steeple? $\sqrt{2.45 \div 39.2} = .25$s. and $.25 \times 10 = 2.5$s. then $2.5 \times 4 = 10$ and $10 \times 10 = 100$ feet, Ans.

3. *To find the depth of a well by dropping a stone into it.*

Rule.—Find the time in seconds to the hearing of the stone strike, by a pendulum; multiply 73088 ($= 16 \times 4 \times 1142$; 1142 feet being the distance sound moves in a second,) by the time in seconds; to this product add 1304164 (=the square of 1142) and from the square root of the sum take 1142; divide the square of the remainder by 64, ($= 16 \times 4$) and the quotient will be the depth of the well in feet; and if the depth be divided by 1142, the quotient will be the time of the sound's ascent, which, taken from the whole time, will leave the time of the stone's descent.

1. Suppose a stone, dropped into a well, is heard to strike the bottom in 4 seconds, what is the depth of the well?

$\sqrt{73088 \times 4 + 1304164} - 1142 = 121.53$, and $121.53 \times 121.53 \div 64 = 230.77$ feet, Ans. Then $230.77 \div 1142 = .2$ of a second, the sound's ascent, and $4 - .2 = 3.8$ seconds, stone's descent.

3. Of the Lever.

It is a principle in mechanics that the power is to the weight as the velocity of the weight is to the velocity of the power.

To find what weight may be balanced by a given power.

Rule.—As the distance between the body to be raised or balanced, and the fulcrum, or prop, is to the distance between the prop and the point where the power is applied, so is the power to the weight which it will balance.

1. If a man weighing 160 lb. rest on a lever 12 feet long, what weight will he balance on the other end, supposing the prop to be 1 foot from the weight? 1 : 11 :: 160 : 1760 lb. Ans.

2. At what distance from a weight of 1440 lb. must a prop be placed, so that a power of 160 lb. applied 9 feet from the prop may balance it? 1440 : 160 :: 9 : 1 foot, Ans.

3. In giving directions for making a chaise, the length of the shafts between the axletree and back band being settled at 9 feet, a dispute arose whereabout on the shafts the centre of the body should be fixed; the chaise-maker advised to place it 30 inches before the axletree; others supposed that 20 inches would be a sufficient incumbrance for the horse. Now supposing two passengers to weigh 3 cwt. and the body of the chaise $\frac{3}{4}$ cwt. more, what will the horse, in both these cases, bear, more than his harness?

Ans. $\begin{cases} 116\frac{2}{3}\text{lb. in the first.} \\ 77\frac{7}{9}\text{lb. in the second.} \end{cases}$

Of the Wheel and Axle.

RULE.—As the diameter of the axle is to the diameter of the wheel, so is the power applied to the wheel to the weight suspended on the axle.

1. If the diameter of the axle be 6 inches, and that of the wheel be 48 inches, what weight applied to the wheel will balance 1268 lb. on the axle? 48 : 6 : : 1268 : 158½lb. Ans.

2. If the diameter of the wheel be 50 inches, and that of the axle 5 inches, what weight on the axle will 2 lb. on the wheel balance? 5 : 50 : : 2 : 20lb. Ans.

3. If the diameter of the wheel be 60 inches, and that of the axle 6 inches, what weight at the axle will balance 1 lb. on the wheel? Ans. 10 lb.

Of the Screw.

The power is to the weight which is to be raised, as the distance between two threads of the screw, is to the circumference of a circle described by the power applied at the end of the lever. *To find the circumference of the circle;* multiply twice the length of the lever by 3.1416; then say, as the circumference is to the distance between the threads of the screw, so is the weight to be raised to the power which will raise it.

1. The threads of a screw are 1 inch asunder, the lever by which it is turned, 30 inches long, and the weight to be raised, 1 ton,= 2240 lb. what power must be applied to turn the screw?

30 × 2 = 60, and 60 × 3.1416 = 188,496 inches, the circumference. Then 188.496 : 1 : : 2240 : 11.88 lb. Ans.

2. If the lever be 30 inches, (the circumference of which is 188 496) the threads 1 inch asunder, and the power 11.88 lb what weight will it raise? 1 : 188.496 : : 11.88 : 2240 lb. nearly, Ans.

3. Let the weight be 2240 lb. the power 11.88 lb. and the lever 30 inches; what is the distance between the threads? Ans. 1 inch, nearly.

4. If the power be 11.88 lb. the weight 2240 lb and the threads 1 inch asunder, what is the length of the lever? Ans. 30 inches nearly.

SECTION V.

MISCELLANEOUS QUESTIONS.

1. What number taken from the square of 48 will leave 16 times 54? Ans. 1440.

2. What number added to the 31st part of 3813 will make the sum 200? Ans. 77.

3. What will 14 cwt. of beef cost, at 5 cents per pound? Ans. \$78.40.

4. How much in length that is $8\frac{3}{8}$ inches wide, will make a square foot? Ans. $17\frac{13}{67}$ inches.

5. What number is that to which if $\frac{2}{7}$ of $\frac{5}{9}$ be added, the sum will be 1? Ans. $\frac{53}{63}$.

6. A father dividing his fortune among his sons, gave A 4 as often as B 3, and C 5 as often as B 6; what was the whole legacy, supposing A's share \$5000? Ans. \$11875.

7. A tradesman increased his estate annually by £100 more than $\frac{1}{4}$ part of it, and at the end of 4 years found that his estate amounted to £10342 3s. 9d.; what had he at first? Ans. £4000.

8. A person being asked the time of day, said the time past noon is equal to $\frac{4}{5}$ of the time till midnight; what was the time? Ans. 20 minutes past 5.

9. The hour and minute hand of a clock are together at 12 o'clock, when are they next together? Ans. 1h. $5\frac{5}{11}$m.

10. A young hare starts 40 yards before a grey hound, and is not perceived by him till she has been up 40 seconds; she scuds away at the rate of 10 miles an hour, and the dog on view makes after it at the rate of 18. In what time and distance will the dog overtake the hare? Ans. $60\frac{5}{22}$s. time. 530yds. distance.

11. What part of 3d. is $\frac{1}{3}$ part of 2d.? Ans. $\frac{2}{9}$.

12. A hare is 50 leaps before a grey-hound, and takes 4 leaps to the grey-hound's 3; but 2 of the grey-hound's leaps are as much as 3 of the hare's; how many leaps must the hound take to catch the hare? If $3 : 1 :: 1 : \frac{1}{3}$ the hare's gain.
$2 : 1 :: 1 : \frac{1}{2}$ the hound's gain.
Then $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ and $\frac{1}{6} : \frac{1}{1} :: \frac{50}{1} : \frac{300}{1} = 300$ Ans.

13. A post is $\frac{1}{4}$ in the sand, $\frac{1}{3}$ in the water, and 10 feet above the water; what is its length? Ans. 24 feet.

14. A man being asked how many sheep he had, said, if he had as many more, half as many more, and $7\frac{1}{2}$ sheep, he should have 20; how many had he? Ans. 5.

15. In an orchard $\frac{1}{2}$ the trees bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plums, and 50 of them cherries; how many trees are there in all? Ans. 600.

16. A can do a piece of work alone in 10 days, B can do it in 13, in what time will both together do it? Ans. $5\frac{15}{23}$ days.

17. What is the difference between the interest of £350 at 4 per cent. for 8 years, and the discount of the same sum at the same rate, and for the same time? Ans. £27 $3\frac{1}{33}$s.

18. Sound moves at the rate of 1142 feet in a second; if the time between the lightning and thunder be 20 seconds, what is the distance of the explosion? Ans. 4.32+ miles.

19. If the earth's diameter be 7911 miles, and that of the moon be 2180; how many moons will be required to make one earth? Ans. 47.788+

20. If a cubic foot of iron were drawn into a bar $\frac{1}{4}$ of an inch square, what would be its length, supposing no waste of metal?

$$\frac{12\times12\times12}{.25\times.25}=27648 \text{ in.}=2304 \text{ ft. Ans.}$$

21. A lent B a solid stack of hay measuring 20 feet every way; some time after, B returned a quantity measuring every way 10 feet; what proportion of the hay is yet due? Ans. $\frac{7}{8}$.

22. A general disposing his army into a square, finds he has 284 soldiers over and above, but increasing each side by one soldier, he wants 25 to fill up the square; how many soldiers had he? Ans. 24000.

23. Supposing a pole 75 feet high to stand on a horizontal plane, at what height must it be cut off, so as that the top of it may fall on a point 55 feet from the bottom, and the end where it was cut off, rest on the stump or upright part?

RULE.—From the square of the length of the pole, (i. e. the sum of the hypothenuse and perpendicular) take the square of the base; then divide the remainder by twice the length of the pole, and the quotient will be the height at which it must be cut off.

$$\frac{\overline{75\times75}-\overline{55\times55}}{75\times2}=17\tfrac{1}{3}\text{ feet, Ans.}$$

24. Suppose a ship sail from lat. 43° N. between N. and E. till her departure from the meridian be 45 leagues, and the sum of her distance and difference of latitude be 135 leagues; what is the distance sailed, and the latitude come to?

$$\frac{\overline{135\times135}-\overline{45\times45}}{135\times2}=\overset{\text{lea.}}{60}=\overset{\text{m.}}{180}=3^\circ \text{ of lat. } 43^\circ+3^\circ=46^\circ \text{ come to.}\quad 135-60=75 \text{l. dis. s'd.}\quad \text{Ans.}$$

25. Four men bought a grindstone 60 inches in diameter; how much of its diameter must each grind off to have an equal share of the stone, if one grind his share first, and then another, till the stone is ground away, making no allowance for the eye?

RULE.—Divide the square of the diameter by the number of men, subtract the quotient from the square, and extract the square root of the remainder, which is the length of the diameter after the first share is taken off, and by repeating the latter part of the process, all the several shares may be found.

$60\times60\div4=900$ the subtrahend.

$\sqrt{3600-900}=51.96+$ and $60-51.96=8.04$ 1st share.

$\sqrt{2700-900}=42.42+$ and $51.96-42.42=9.54$ 2d share.

$\sqrt{1800-900}=30.$ and $42.42-30=12.42$ 3d share.

and 30 4th's share.

26. Suppose one of those meteors called fireballs to move parallel to the earth's surface, and 50 miles above it, at the rate of 20 miles per second; in what time will it move round the earth?

The earth's diameter being 7964 miles, the diameter of the orbit will be $7964+\overline{50\times2}=8064$ and $8064\times3.1416=25333.8624$ its circumference. Then $25333.8624\div20=1266.69312$s. $=21'$ $6''$ $41'''$ $35''''$ $13'''''$ $55''''''$ Ans.

27. When first the marriage knot was tied betwixt my wife and me,
My age with her's did so agree as nineteen does with eight and three;
But after ten and half ten years, we man and wife had been,
Her age came up as near to mine as two times three to nine.

What were our ages at marriage? Ans. 57 and 33.

28. A body weighing 30lb. is impelled by such a force as to send it 20 rods in a second; with what velocity would a body weighing 12lb. move if it were impelled by the same force? Ans. 50 rods.

29. In a thunder storm I observed by my clock that it was 6 seconds between the lightning and thunder; at what distance was the explosion? Ans. 6852ft.$=1\frac{1}{4}\frac{3}{4}\frac{1}{0}$ miles.

30. I have a square stick of timber 18 inches by 14, but one with a third part of the timber in it, provided it be 8 inches deep, will serve; how wide will it be? Ans. $10\frac{1}{2}$ inches.

31. There is a square pyramid, each side of whose base is 30 inches, and whose perpendicular height is 120 inches, to be divided into three equal parts by sections parallel to its base; what will be the perpendicular height of each part?

$30\times30\times40=36000$ the solidity in inches. Now $\frac{2}{3}$ of this is 24000, and $\frac{1}{3}$ is 12000. Therefore, as $36000 : 120\times120\times120 ::$ $\left\{\begin{matrix}24000 : 1152000\\12000 : 576000\end{matrix}\right\}$ Then, $\sqrt[3]{1152000}=104.8$. Also, $\sqrt[3]{576000}=83.2$. Then $120-104.8=15.2$, length of the thickest part, and $104.8-83.2=21.6$, length of the middle part; consequently, 83.2 is the length of the top part.

32. There are 4 spheres, each 4 inches in diameter, lying so as to touch each other, in the form of a square, and on the middle of this square is put a fifth ball of the same diameter; what is the distance between the two horizontal planes passing through the centres of the balls?

$\sqrt{4^2+4^2\div2}=2.828+$ inches, Ans.

33. There are two balls, each 4 inches in diameter, which touch each other, and another ball of the same diameter is so placed between them that their centres are in the same vertical plane; what is the distance between the horizontal planes which pass through their centres?

$\sqrt{4^2-2^2}=3.46+$ inches, Ans.

34. A military officer drew up his soldiers in rank and file, having the number in rank and file equal; on being reinforced with three times his first number of men, he placed them all in the same form, and then the number in rank and file was just double what it was at first; he was again reinforced with three times his number of men, and after placing the whole in the same form as at first, his number in rank and file was 40 men each; how many men had he at first? Ans. 100.

35. If a weight of 1440 lb. be placed 1 foot from the prop, at what distance from the prop must a power of 160 lb. be applied to balance it? Ans. 9 feet.

36. Tubes may be made of gold weighing not more than at the rate of $\frac{1}{1625}$ of a grain per foot; what would be the weight of such a tube, which would extend across the Atlantic from Boston to London, estimating the distance at 3000 miles?

Ans. 1 lb. 8 oz. 6 pwt. $3\frac{9}{13}$ gr.

37. Divide 1000 crowns; give A 129 more than B, and B 178 fewer than C. Ans. A 360, B 231, and C 409.

38. A person dying, left his wife with child, and by his will ordered that if she went with a son $\frac{2}{3}$ of the estate should belong to him, and the remainder to his mother; and if she had a daughter, he appointed the mother $\frac{2}{3}$ and the daughter $\frac{1}{3}$; but it happened she was delivered of both a son and a daughter; by which she lost in equity £2000 more than if it had been only a girl; what would have been her dowry if she had only a son? Ans. £1750.

39. A tradesman increased his estate annually a third, abating £100 which he usually spent in his family, and at the end of $3\frac{1}{4}$ years, found that his net estate amounted to £3154 11s. 8d.; what had he at outsetting? Ans. £1411 12s. $9\frac{3}{4}$d.

40. Three persons enter joint trade together, to which A contributed £210, B £312, they clear £140, whereof £37 10s belongs of right to C, that person's stock and the several gains of the other two are required.

Ans. C's stock £190 19s. 6d. A gained £41 4s. $8\frac{1}{2}$d.

41. A, B and C will trench a field in 12 days, B, C and D in 14, C, D and A will do it in 15, and D, A, B, in 18; in what time will it be done by all of them together; and by each of them singly?

Ans. together in 10.83 days, by A 47.848, B in 38.969, C in 27.194, D in 111.176 days.

SECTION VI.

Book-Keeping.

Book-Keeping is the method of recording a systematick account of mercantile transactions.

Every mercantile transaction consists is giving one thing for another. This change of property should be distinctly recorded in a book, or books prepared for the purpose, so that the man of business may at all times know the true state of his affairs.

FARMERS' BOOK-KEEPING.

FIRST METHOD.

By this method but one book is necessary, which should be ruled with four columns on the right hand side of each page, two for debtor* columns, and two for credit, and one column on the left hand side for the date, as in the following example.

1825.	*THOMAS HARDY,*	Debtor. $	cts.	Creditor. $	cts.
Jan. 28.	Dr. to 2½ tons of hay, at $8.	20	00		
" 29.	Cr. by 14 bush. of corn, at 48 cts.			6	72
Feb. 2.	Cr. by cash,			5	00
" 4.	Dr. to 30lb. of flax, at 12 cts.	3	60		
" 9.	Dr. to 25lb. of flax, at 12 cts.	3	00		
April 14.	Cr. by 12 bush. of wheat, at $1.			12	00
" "	Cr. by cash to balance.			2	88
		26	60	26	60

On account of its simplicity, the above method is probably the best which can be recommended to farmers and country mechanics. In keeping books in this way, it will be necessary to leave a considerable blank after each man's account, that it may be continued without transferring it to another part of the book; and also to have a list of the names with the page standing against it, for the more convenient reference to the several accounts.

SECOND METHOD.

By this method the debt and credit are entered on separate pages facing each other, with the debt on the left hand, and the credit on the right hand, as in the following example.

*The person who receives any thing of me is *Dr.* to me, and the person from whom I receive is *Cr.* Or, the person who becomes indebted to me, where by receiving goods or money, or by my paying his debts, &c. he must be entered Dr.: and the person to whom I become indebted, whether by receiving from him goods or money, or by the payment of my debts, must be entered Cr.

1825.	PETER PINDLE, Dr.	$	cts	1825.	PETER PINDLE Cr	$	cts.
Jan. 1.	To 3 cords of wood, at $1 50	4	50	Jan. 1.	By 12lb. shingle-nails, at 10 cts.	1	20
8.	To 5½ bush. of rye, at 50 cts.	2	75	6.	By 25lb. of sugar, at 11 cts.	2	75
Feb. 2.	To 3 bush. of wheat, at $1 25	3	75	21.	By 1½ cwt. iron, at $6	9	00
14	To 5 cords of wood, at $1 50	7	50	Feb. 11.	By 2 lb. young hyson tea, at $1 10	2	20
19.	To 7 bush. of oats, at 25 cts.	1	75	13	By 10lb. of loaf sugar at 30 cts.	3	00
24.	To cash, to balance	3	30	24.	By 6 yds. black silk, at 90 cts.	5	40
		23	55			23	55

Either of the foregoing methods may answer for farmers and for country mechanics generally, but to the retail merchant and others whose business is extensive, an acquaintance with book-keeping by the day-book, and ledger called *single entry*, or by the day-book, journal and ledger, called *double entry*, is indispensible. The latter is much the most perfect system, and far best for wholesale dealers, but as it is more complicated and seldom used, we shall confine our attention to the former, which is generally adopted by merchants and others in this country.

BOOK-KEEPING BY SINGLE ENTRY.

Single Entry requires two principal books, the day-book, or waste book, and the ledger, and one auxiliary book, the cash-book.

1. THE DAY-BOOK.

This book is ruled with two columns on the right hand for dollars and cents, one column on the left for inserting the folio or page of the ledger, to which the account is transferred, and a top line over which is written the month, date and year. The articles are separated from each other by a line drawn across the page, and the transactions of one day from those of another by a double line in the centre of which is the day of the month.

This book commences with an account of all the property, debts, &c. of the person, and is followed by a distinct record of all the transactions in trade in the order of time in which they occur, with every circumstance necessary to render the transaction plain and intelligible.*

In entering accounts in the day-book, the following order should be observed: 1, the date; 2, the name of the person, with the abbreviation Dr. or Cr. at the right hand as he is debtor or creditor, by the transaction;† 3, the article or articles with the price annexed, and the value carried out into the ruled columns, with the amount

*As the day-book is the decisive book of reference, in case of any supposed mistake, or error, in the accounts in the ledger, it is of the greatest importance that every transaction be noted in it with particular perspicuity and accuracy.

† To know when a person is to be entered Dr. and when Cr. see note * preceding page.

placed directly under, when there is more than one article charged; and 4, the page to which the account is transferred in the ledger. For the better understanding of the day-book, see the specimen annexed.

2. THE LEDGER.

Each page of the ledger is ruled with a top line, on which is written the name of the person, and marked *Dr.* on the left hand for receiving the debited articles, and *Cr.* on the right for receiving the credited articles of the day-book. On the right hand of both Dr. and Cr. sides, are ruled two columns for dollars and cents, and on their left, three columns, one for the page of the day-book, one for the month, and one for the date. The ledger has an index, in which the names of persons are arranged under their initial letters, with the page in the ledger, where the account may be found.

Rule for Posting.—Under the name of the person, enter the several transactions on the Dr. or Cr. side in the ledger, as they stand debited or credited in the day-book. When several things are included in the same transaction, they are distinguished by the term "sundries." Some accountants enter in the ledger only the page of the day-book and the amount of the transaction, without specifying the items, but the former is thought the most correct.

Balancing Accounts.—When all the articles are correctly posted into the ledger, each account is balanced by subtracting the less side from the greater, and entering the balance on the less side, by which both sides are made equal. The excess of all the balances on the Dr. over those on the Cr. sides, added to the cash on hand and the value of the goods unsold, the sum is the net of the estate, which, compared with the stock at the commencement of business, exhibits the merchant's gain.*

3. CASH-BOOK.

In the cash-book are recorded the daily receipt and payment of money. For this purpose it is ruled with separate columns, one for money received and the other for money paid, in which should be recorded merely the date, to or by whom paid and the sum. The cash-book is convenient, but not absolutely necessary. Other auxiliary books are sometimes used and are important in some kinds of business, but the accountant will readily form these for himself, as circumstances may render necessary.

* When the place assigned to any person's account is filled with items, the person's name must not be entered the second time, but may be transferred to another page in the following manner, viz. Add up the Dr. and Cr. columns and against the sums with *Amount transferred to page* —, here inserting the page where the new account is opened. Begin the new account by entering on the Dr. side, *To amount brought from page* —, inserting the page of the old account, and on the Cr. side, *By amount brought from page* —, inserting the page of the old account, placing the sums in their proper columns.

As several day books and ledgers may be necessary in the progress of business, they should be distinguished by lettering them, as follows: day-book A. day-book B, &c.—Ledger A. ledger B, &c. and in posting accounts into the ledger, there must be a reference to day-book A or B, &c. as the account is found in one or another.

DAY-BOOK.

1] Albany, January 3, 1825.

L. P.			$	ct.
	INVENTORY			
	Of ready money, goods and debts due to me, Timothy Standish, Merchant, Albany.			
	Money on hand, - -	$823.00		
1	Peter Pindar owes me,	212.00		
1	John Kelley, - - -	- 122.00		
2	Thomas Scott, - -	16.00		
	16 cwt. Sugar, a $9.50	- 152.00		
	25 quintals Fish, a 3.50 -	87,50		
	300 lbs. Coffee, a $18. -	- 44 00		
		1456.50		
	DEBTS			
	Owed by me, the said Timothy Standish.			
2	To David Terry, as per account, - - - -	$12,00		
2	" John Strong, - - -	- 146,00		
2	" Felix Storrs, - -	238,00		
		396,00		
		Net	1060	50
	—4—			
1	Samuel English	Dr.		
	To 2 quintals fish, - -	a $4,25		
	" 20 lbs. coffee - -	a ,22	12	90
1	Peter Pindar	Cr.		
	By Cash on former account,		112	
	—7—			
1	Sylvester Warren	Dr.		
	To 48 lbs. sugar - -	a ,12		
	" 7 lbs. coffee - - -	a ,22	7	30
	—10—			
1	Samuel Adams	Cr.*		
	By 2 chests Hyson tea, 160 lbs. - - - -	a $1.00		
	4 chests Bohea tea, 320 lb.	a ,40	288	
1	Levi Munson	Dr.		
	To 3 lbs. Bohea Tea - -	a ,62		
	1 lb. Hyson Tea -	a $1.25		
	4 lbs. Coffee - - -	a ,22		
	10 lbs. Sugar - - -	a ,12	5	19
	—13—			
1	Zera Coleman	Dr.		
	To 3 quintals Fish - -	a $4,24	12	75
1	John Kelley	Cr.		
	By Cash, in account,		50	
2	John Strong	Dr.		
	To Cash in former account,		46	75
	—24—			
1	Charles Gray	Dr		
	To 8 lbs. Sugar - -	a .12		
	4 lbs. Coffee - - -	a ,22		
	3 lbs. Hyson Tea -	a $1,25	5	59

February 2 [2

L.P.			$	ct
	Titus Cole	Cr.		
2	By 120 gal. Molasses - -	a ,28		
	86 gal. wine - -	a $1,31		
	116 gal N. E. Rum - -	a ,42	194	93
1	Simon Pond	Dr.		
	To 5 gal. N. E.	3	2	65
2	Calvin Owen	Dr.		
	To 1 gal. Wine - -	a $1,75		
	7 gal. Molasses - -	a ,42	4	69
1	Samuel Adams	Dr.		
	To Cash on account,		126	75
	—5—			
1	Samuel English	Cr.		
	By 6 bush. Wheat - -	a 83	4	98
2	Thomas Scott	Cr.		
	By Cash to balance		16	
	—8—			
1	Levi Munson	Dr.		
	To 4 quintals Fish - -	a $4,00		
	40 lbs. Sugar - - -	a ,12		
	5 gal. Molasses - -	a ,42	22	90
		Cr.		
1	By Cash, on account, - -	$10,00		
	8 bush. Corn - -	a ,48		
	10 bush. Rye - - -	a .50	18	84
1	John Kelley	Cr.		
	By Cash on account, to bal.		72	
	—10—			
2	Dan Burt	Dr.		
	To 10 gal. N. E. Rum - -	a ,50		
	5 gal. Molasses . -	,40	7	
2	Philip Carter	Dr.		
	To 16 lbs. Coffee - - -	a ,22		
	12 lbs. sugar - - -	a ,12		
	4 lbs. Bohea Tea - -	a ,61		
	1 quintal Fish - -	a $4,25	11	65
	—11—			
2	John Dana	Dr.		
	To 4 gal. Wine -	a $1,25	7	00
	—12—			
2	David Ferry	Dr.		
	To Cash to balance former act.		12	
1	Peter Pindar	Cr.		
	By Cash in full, - - -	-	100	
2	Felix Storrs	Dr.		
	To Cash on former account,		138	
	—14—			
2	David French	Dr.		
	To 2 quintals Fish - -	a $4,25	8	50
1	Samuel English	Cr.		
	By 10 bushels Rye - -	a ,54		
	Cash to balance - -	$2,52	7	92

*By single entry, goods bought are entered, either in an invoice book kept for the purpose, or posted immediately into the Leger from the invoices, or bills of parcels. This method is now, however, adopted here: but the goods are credited the seller, and afterwards to his account in the Leger.

DAY-BOOK.

3] February 14. March 11. [4

L.P.			$	ct.
	Sylvester Warren	Dr.		
1	To 1 gal. Wine - -	a $1,75		
	3 gal. N. E. Rum - -	a ,53	3	34
		Cr.		
1	By 10 bushels Wheat - -	a ,92		
	3 bu. corn - - - -	a 48	10	64
2	John Strong	Dr.		
	To Cash to balance former ac.		99	25
	16			
2	Aaron Potter	Dr.		
	To 24 lbs Hyson Tea -	a $1,20		
	2 1-2 quintals Fish - -	a 4,16		
	50 lbs Coffee - -	a ,20	49	05
1	Zera Coleman	Cr.		
	By 233 lbs pork - -	a ,04 1-2	10	48
	20			
1	Titus Cole	Dr.		
	To Cash in full, - - -		194	98
	26			
1	Simon Pond	Dr.		
	To 1 chest Bohea Tea, 80 lbs	a ,44	35	20
1	Samuel Adams	Dr.		
	To Cash in full, - -		161	52
	March 1			
2	Jared Hill	Dr.		
	To 21 3-4 lbs Coffee - -	a ,22		
	36 1-4 lbs Sugar - -	a ,14		
	9 1-2 gal. Wine -	a $1,62	24	25
2	Charles Lyman	Dr.		
	To 6 1-2 quintals Fish -	a $4,25	27	63
	4			
2	Dan Burt	Cr.		
	By Cash in full. By J. Starr.		7	
	5			
1	Simon Pond	Dr.		
	To 4 quintals of Fish -	a 4,25	17	
	7			
1	Charles Gray	Cr.		
	By 5 1-2 bushels wheat -	a ,92		
	Cash to balance - -	,53	5	59
2	Augustus Young	Dr.		
	To 112 1-2 lbs Sugar - -	a ,11	12	38
2	Calvin Owen	Dr.		
	To 5 3-4 lbs Hyson Tea -	a $1,19	6	84
	8			
2	Noah Drew	Cr.		
	By 1 hhd. W. I. Rum, 63 gal.	a ,75	47	25
	10			
2	Calvin Owen	Cr.		
	By Cash in full - - - -		11	53
1	Levi Munson	Cr.		
	By cash on account - - -		5	
	11			
1	Charles Gray	Dr.		
	To 10 gal. W. I. Rum -	a $1,25	12	50

L.P.			$	ct.
	Levi Munson	Dr.		
1	To 5 1-2 gal. of W. I Rum	a $1,25		
	16 gal. Molasses - -	a ,40	13	28
	14			
2	Felix Storrs	Dr.		
	To Cash in full - - - -		100	
	15			
2	Philip Carter	Cr.		
	By an order on John Tinker for		11	65
2	Aaron Potter	Cr.		
	Br Cash on account - - -		25	50
	18			
1	Simon Pond	Cr.		
	By 21 1-2 bush. Rye - -	a ,52		
	11 1-2 bush. Corn - -	a ,48	16	70
	19			
1	Levi Munson	Dr.		
	To 12 gal. N. E. Rum - -	a ,50	6	
	22			
2	John Dana	Cr.		
	By Cash in full - - -		7	
2	Charles Lyman	Cr.		
	By Cash, in full, on account -		27	63
	24			
2	David French	Dr.		
	To 1 1-2 gal. wine - -	a $1,75		
	3 gal. W. I. Rum - -	a ,94	5	45
	26			
2	Jared Hill	Cr.		
	By Cash in full on acct. - -		24	25
2	Noah Drew	Dr.		
	To 233 lbs pork - -	a ,05		
	10 bush. Wheat - -	a ,98	21	45
	28			
1	Levi Munson	Dr.		
	To 16 lbs Coffee - -	a ,22		
	4 " Hyson Tea -	a $1,20	8	32
	30			
2	David French	Cr.		
	By Cash in full - - - -		13	95
	31			
2	Augustus Young	Dr.		
	To 13 lbs Coffee - -	a ,22	2	86
		Cr.		
2	By 10 3-4 bush. Wheat -	a ,94		
	Cash to balance - -	$5,29	15	24
	April 2			
1	Levi Munson	Cr.		
	By Cash on account - - -		10	25
1	Charles Gray	Cr.		
	By Cash on acct. in full - -		12	50
1	Simon Pond	Dr.		
	To 28 gals. N. E. Rum -	a ,51		
	26 1-2 gals. W. I Rum	a ,94	38	70

1] LEGER. [1

Dr. SAMUEL ENGLISH **Cr.**

1825	D.B. F.		$	ct	1825	D.B. F.		$	
Jan. 4	1	To Sundries as per D. Book,	12	90	Feb. 5	2	By 6 bush. Wheat	4	98
					" 13		Sundries	7	92
								12	90

Dr. PETER PINDAR **Cr.**

1825					1825				
Jan. 3	1	To balance on old account	212		Jan. 4	1	By Cash on account	112	
					Feb. 12	2	Cash in full	100	
								212	

Dr. SYLVESTER WARREN **Cr**

1825					1825				
Jan. 7	1	To Sundries	7	30	Feb. 14	4	By Sundries	10	64
Feb. 14	3	Sundries	3	34					
			10	64					

Dr. SAMUEL ADAMS **Cr.**

1825					1825				
Feb. 3	2	To Cash, on account	126	75	Jan. 10	1	By Sundries	288	
26	3	Cash in full	161	25					
			288	00					

Dr. LEVI MUNSON **Cr.**

1825					1825				
Jan. 10	1	To Sundries	5	19	Feb. 8	2	By Sundries	18	84
Feb. 8	2	Sundries	22	90	March 10	3	Cash on account	5	
		Amount transferred, page 1	28	09			Amount transferred, page 1	23	84

Dr. CHARLES GRAY **Cr.**

1825					1825				
Jan. 24	1	To Sundries	5	59	March 7	3	By Sundries	5	59
March 11	3	10 gals. W. I. Rum	12	50	April 2	4	Cash in full	12	50
			18	09				18	09

Dr. SIMON POND **Cr.**

1825					1825				
Feb. 2	2	To 5 gals N. E. Rum	2	65	March 18	4	By Sundries	16	70
26	3	1 chest Bohea Tea	35	20			Balance, transferred	76	85
March 5	3	4 quintals Fish	17					93	55
April 2	4	Sundries	38	70					
			93	55					

Dr. LEVI MUNSON **Cr.**

1825					1825				
March 11	3	To amount brought from page 1	28	09	April 2		By amt. br. from page 1	23	84
" 19	4	Sundries	13	28		4	Cash on account	10	25
" 28	4	12 gals N. E. Rum	6				Balance, transferred,	21	60
		Sundries	8	32				55	69
			55	69					

Dr. ZERA COLEMAN **Cr.**

1826					1825				
Jan. 13	1	To 3 quintals Fish	12	75	Feb. 16	3	By 233 lbs Pork	10	48
							Balance, transferred,	2	27
								12	75

Dr. JOHN KELLEY **Cr.**

1825					1825				
Jan. 3	1	To balance on old account	122		Jan. 13	1	By Cash on account	50	
					Feb. 8	2	Cash in full	72	
								122	

Dr. TITUS COLE **Cr**

1825					1825				
Feb. 20	3	To Cash in full	194	98	Feb. 2	2	By Sundries	194	98

[2 **LEGER.** [2

Dr. JOHN STRONG **Cr.**

1825					1825				
Jan. 13	1	To Cash on account	46	75	Jan. 3	1	By balance on old acct.	146	
Feb. 14		Cash in full	99	25					
			146	00					

Dr. CALVIN OWEN **Cr.**

1825					1825				
Feb. 3	2	To Sundries	4	69	March 10	4	By Cash in full	11	53
March 7	3	5 3-4 lbs Hyson Tea	6	84					
			11	53					

Dr. THOMAS SCOTT **Cr.**

1825					1825				
Jan. 3	1	To balance on former account	16		Feb. 5	2	By Cash in full	16	

Dr. DAN BURT **Cr.**

1825					1825				
Feb. 10	2	To Sundries	7		March 4	3	By Cash in full	7	

Dr. PHILIP CARTER **Cr.**

1825					1825				
Feb. 10	2	To Sundries	11	65	March 15	4	By order on J.Tinker for	11	65

Dr. JOHN DANA **Cr.**

1825					1825				
Feb. 11	2	To 4 gals of Wine	7		March 22	4	By Cash in full	7	

Dr. DAVID TERRY **Cr.**

1825					1825				
Jan. 3	1	To balance on old account	12		Feb. 12	2	By Cash in full	12	

Dr. FELIX STORRS **Cr.**

1825					1825				
Feb. 12	2	To Cash on account	138		Jan. 3	1	By balance on old act.	238	
March 14	4	Cash in full	100						
			238						

Dr. DAVID FRENCH **Cr.**

1825					1825				
Feb. 14	2	To 2 quintals of Fish	8	50	March 30	4	By Cash in full	13	95
March 24	4	Sundries	5	45					
			13	95					

Dr. AARON POTTER **Cr.**

1825					1825				
Feb. 16	3	To Sundries	49	05	March 15	4	By Cash on account	25	50
							Balance	23	55
								49	15

Dr. CEARLES LYMAN **Cr**

1825					1825				
March 7	3	To 6 1-2 quintals Fish	27	63	March 22	4	By Cash in full	27	63

Dr. AUGUSTUS YOUNG **Cr.**

1825					1825				
March 7	3	To 112 1-2 lbs Sugar	12	38	March 31	4	By Sundries	15	24
31	4	13 lbs Coffee	2	86					
			15	24					

Dr. JARED HILL **Cr**

1825					1825				
March 1	3	To Sundries	24	25	March 26	4	By Cash in full	24	25

Dr. NOAH DREW **Cr**

1825					1825				
March 26	4	To Sundries	21	45	March 8	3	By 1 hhd. W I. Rum	47	25
		Balance	25	80					
			47	25					

Index to the Leger.

	P				
A		**G**		**P**	
Adams, Samuel	1	Gray, Charles .	1	Pindar, Peter .	1
B		**H**		Pond, Simon .	1
Burt, Dan .	2	Hill, Jared .	2	Petter, Aaron .	2
C		**K**		**S**	
Carter, Philip .	2	Kelly, John .	1	Scott, Thomas .	2
Cole, Titus .	1			Storrs, Felix .	2
Coleman, Zerah	1	**L**		Strong, John .	2
D		Lyman, Charles	2	**T**	
Dana, John .	2			Terry, David .	2
Drew, Noah .	2	**M**		**W**	
E		Munson, Levi .	1	Warren, Sylvester	1
English, Samuel	1			**Y**	
F		**O**		Young, Augustus	2
French, David .	2	Owen, Calvin .	2		

Book-Keeping by Single Entry, shows clearly the state of accounts with individuals dealing on credit, but does not exhibit the true state of his affairs to the book keeper himself. If therefore he wishes to know his profits or losses by his business, he must take an inventory of his stock on hand, balances on book and ready money, and this inventory compared with that taken at the commencement of business, will show the gain or loss by trade.

Inventory taken from the foregoing example; April 3, 1825.

Money on hand .	\$468.54	Brought up	994.39
$148\frac{1}{4}$ lb. Coffee, *a* 18	27.01	Produce on hand .	50.75
$1333\frac{1}{4}$ lb. Sugar, *a* $09\frac{1}{2}$	126.66	Due me as per Leger	124.27
$122\frac{1}{4}$ lb. H. Tea, *a* \$1	122.25		
233 lb. Bohea Tea, *a* .40	93.20		1169.41
$23\frac{1}{2}$ g. W.I. Rum, *a* .75	17.62	I owe as per Leger	25.80
58 gals. N.E. Rum, *a* .42	24.36		
87 gals. Molasses, *a* .28	24.36	Net Estate, Ap. 3, 1825	1143.61
69 gals. Wine, *a* \$1.31	90.39	Net Estate, Jan. 3, 1825	1060.50
Carried up	994.39	Net gain in 3 months	\$83.11

2. BILLS OF PARCELS.

No. I.

Monrovia, Jan. 25, 1825.

Mr. Oliver Durance,

Bought of Mr. George Merchant,

8yds. of Camblet, at 5	\$6.67
3yds. of Bocking, at 3/6 -	1.75
3yds. of Bombazett, at 2/3	1.12
$\frac{5}{16}$yd. of Plush, at 10/6	0.55
	\$10.09

Charged in account.

No. II.

Peru, Dec. 29, 1824.

Mr. Mason Prior,

Bought of John Lurcher,

One pair of Oxen -	\$67.00
Four Cows - -	49.50
Received payment,	\$116,50

John Lurcher.

3. OF NOTES.

No. I.

Overdean, Sept. 17, 1822.—For value received I promise to pay to *Oliver Bountiful*, or order, sixty-three dollars, fifty-four cents, on demand, with interest after three months. WILLIAM TRUSTY.

Attest, *Timothy Testimony.*

No. II.

Billfort, Sept. 17, 1822.—For value received, I promise to pay to O. R. or bearer, ——— dollars —— cents, three months after date.

PETER PENCIL.

No. III. *By two Persons.*

Arian, Sept. 17, 1822.—For value received, we jointly and severally promise to pay to C. D. or order, ——— dollars —— cents on demand with interest. ALDEN FAITHFUL.

Attest, *Constance Adley.* JAMES FAIRFACE.

4. OF BONDS.

A Bond with a Condition from one to another.

KNOW all men by these presents, that I, C. D. of &c. in the county of &c. am held and firmly bound to E. F. of &c. in two hundred dollars, to be paid to the said E. F. or his certain attorney, his executors, administrators, or assigns, to which payment, well and truly to be made, I bind myself, my heirs, my executors and administrators, firmly by these presents. Sealed with my seal. Dated the eleventh day of ——— in the year of our Lord one thousand eight hundred and twenty-two.

The condition of this obligation is such, that if the above bound C. D. his heirs, executors, or administrators, do and shall well and truly pay, or cause to be paid unto the above named E. F. his executors, administrators, or assigns, the full sum of two hundred dollars, with legal interest for the same, on or before the eleventh day of ——— next ensuing the date hereof: then this obligation to be void, or otherwise to remain in full force and virtue.

Signed, &c.

A Condition of a Counter Bond, or Bond of Indemnity, where one man becomes bound for another.

THE condition of this obligation is such, that whereas the above named A. B. at the special instance and request, and for the only proper debt of the above bound C. D. together with the said C. D. is, and by one bond or obligation bearing equal date with the obligation above written, held and firmly bound unto E. F. of &c. in the penal sum of ——— dollars, conditioned for the payment of the sum of &c. with legal interest for the same, on or before the —— day of —— next ensuing the date of the said in part recited obligation, as in and by the said in part recited bond, with the condition thereunder written, may more fully appear: If, therefore, the said C. D. his heirs, executors, or administrators, do and shall well and truly pay or cause to be paid unto the said E. F. his executors, administrators, or assigns, the said sum of &c. with legal interest of the same, on the said ——— day of, &c. next ensuing the date of the said in part recited obligation, according to the true intent and meaning, and in full discharge and satisfaction of the said in part recited bond or obligation: Then, &c. otherwise, &c.

5. OF RECEIPTS.

No. I.

Sitgrieves, Sept. 19, 1824. Received of Mr. *Durance Adley*, ten dollars in full of all accounts. ORVAND CONSTANCE.

No. II.

Sitgrieves, Sept. 19, 1824. Received of Mr. *Orvand Constance*, five dollars in full of all accounts. DURANCE ADLEY.

No. III. Receipt for an endorsement on a note.

Sitgrieves, Sept. 19, 1824. Received of Mr. Simpson Eastly, (by the hand of Titus Trusty) sixteen dollars twenty-five cents, which is endorsed on his note of June 3, 1820. PETER CHEERFUL.

No. IV. A Receipt for money received on account.

Sitgrieves, Sept. 19, 1824. Received of Mr. Orand Landike, fifty dollars on account. ELDRO SLACKLEY.

No. V. Receipt for interest due on a bond.

Received this ——— day of ——— of Mr. A. B. the sum of five pounds in full of one year's interest of £100 due to me on the ——— day of ——— last, on bond from the said A. B. I say received,

By me, C. D.

6. OF ORDERS.

No. I.

Mr. Stephen Burgess,—SIR,

For value received, pay to A. B. ten dollars, and place the same to my account. SAMUEL SKINNER.

Archdale, Sept. 9, 1824.

No. II.

SIR, *Boston, Sept.* 9, 1824.

For value received, pay G. R. eighty-six cents, and this with his receipt shall be your discharge from me. NICHOLAS REUBENS.

To Mr. James Robottom.

7. OF DEEDS.

No. I. A Warrantee Deed.

KNOW ALL MEN BY THESE PRESENTS, That I, Peter Careful, of Bridgewater, in the County of Windsor, and State of Vermont, gentleman, for and in consideration of one hundred and fifty dollars, and forty-five cents, paid to me by Samuel Pendleton, of Woodstock, in the County of Windsor, and State of Vermont, yeoman, the receipt whereof I do hereby acknowledge, do hereby give, grant, sell and convey to the said Samuel Pendleton, his heirs and assigns, a certain tract and parcel of land, bounded as follows, viz.

[*Here insert the bounds, together with all the privileges and appurtenances thereunto belonging.*]

To have and to hold the same unto the said Samuel Pendleton, his heirs and assigns to his and their use and behoof forever. And I do covenant with the said Samuel Pendleton, his heirs and assigns, that I am lawfully seized in fee of the premises, that they are free of all incumbrances, and that I will warrant and defend the same to the said Samuel Pendleton, his

heirs and assigns forever, against the lawful claims and demands of all persons.

In witness whereof, I hereunto set my hand and seal, this —— day of —— in the year of our Lord one thousand eight hundred and twenty.

Signed, sealed, and delivered in presence of PETER CAREFUL. O

L. R. F. G.

No. II. *Quitclaim Deed.*

KNOW ALL MEN BY THESE PRESENTS, That I, A. B. of, &c. in consideration of the sum of —— dollars, to me paid by C. D. of, &c. the receipt whereof I do hereby acknowledge, have remised, released, and forever quitclaimed, and do by these presents remit, release, and forever quitclaim unto the said C. D. his heirs and assigns forever [*Here insert the premises.*] To have and to hold the same, together with all the privileges and appurtenances thereunto belonging, to him the said C. D. his heirs and assigns forever.

In witness, &c.

No. III. *Mortgage Deed.*

KNOW ALL MEN BY THESE PRESENTS, That I Simpson Easley, of —— in the County of —— in the State of —— Blacksmith, in consideration of —— Dollars —— Cents, paid by Elvin Fairface of —— in the County of —— in the State of —— Shoemaker, the receipt whereof I do hereby acknowledge, do hereby give, grant, sell and convey unto the said Elvin Fairface, his heirs and assigns, a certain tract and parcel of land, bounded as follows, viz. (*Here insert the bounds, together with all the privileges and appurtenances thereunto belonging.*) To have and to hold the afore granted premises to the said Elvin Fairface, his heirs and assigns, to his and their use and behoof forever. And I do covenant with the said Elvin Fairface, his heirs and assigns, that I am lawfully seized in fee of the afore granted premises. That they are free of all incumbrances: That I have good right to sell and convey the same to the said Elvin Fairface. And that I will warrant and defend the same premises to the said Elvin, his heirs and assigns forever, against the lawful claims and demands of all persons. *Provided nevertheless*, That if I the said Simpson Easley, my heirs, executors, or administrators shall well and truly pay to the said Elvin Fairface, his heirs, executors, administrators or assigns, the full and just sum of —— dollars —— cents, on or before the —— day of —— which will be in the year of our Lord eighteen hundred and —— with lawful interest for the same until paid, then this deed, as also a certain bond [*or note as the case may be*] bearing even date with these presents, given by me to the said Fairface, conditioned to pay the same sum and interest at the time aforesaid, shall be void, otherwise to remain in full force and virtue. In witness whereof, I the said Simpson and Abigail my wife, in testimony that she relinquishes all her right to dower or alimony in and to the above described premises, hereunto set our hands and seals this —— day of —— in the year of our Lord one thousand eight hundred and twenty-five.

Signed, sealed, and delivered in presence of SIMPSON EASLEY. O
ABIGAIL EASLEY. O

L. N. V. X.

8. OF AN INDENTURE.

A common Indenture to bind an Apprentice.

THIS Indenture witnesseth, That A. B. of, &c. hath put and placed, and by these presents doth put and bind out his son C. D. and he the said C. D, doth hereby put, place and bind out himself, as an apprentice to R. P. to learn the art, trade, or mystery of —— The said C. D. after the manner of an apprentice, to dwell with and serve the said R. P. ——— from the day of the date hereof, until the ——— day of ——— which will be in the year of our Lord one thousand eight hundre — at which time the said apprentice, if he should be living, will be tw ne years of age. During which time or term the said apprentice his said master well and faithfully shall serve; his secrets keep, and his lawful commands every where and at all times readily obey. He shall do no damage to his said master, nor wilfully suffer any to be done by others; and if any to his knowledge be intended, he shall give his master seasonable notice thereof. He shall not waste the goods of his said master, nor lend them unlawfully to any; at cards, dice, or any unlawful game he shall not play; fornication he shall not commit, nor matrimony contract during the said term; taverns, alehouses, or places of gaming he shall not haunt or frequent: From the service of his said master he shall not absent himself; but in all things, and at all times he shall carry and behave himself as a good and faithful apprentice ought, during the whole time or term aforesaid.

And the said R. P. on his part doth hereby promise, covenant and agree to teach and instruct the said apprentice, or cause him to be taught and instructed in the art, trade, or calling of a —— by the best way or means he can, and also teach and instruct the said apprentice, or cause him to be taught and instructed to read, write, and cipher as far as the Rule of Three, if the said apprentice be capable to learn, and shall well and faithfully find and provide for the said apprentice, good and sufficient meat, drink, clothing, lodging and other necessaries fit and convenient for such an apprentice, during the term aforesaid, and at the expiration thereof, shall give unto the said apprentice, two suits of wearing apparel, one suitable for the Lord's day, and the other for working days.

In testimony whereof, the said parties have hereunto interchangeably set their hands and seals, this said —— day of ——— in the year of our Lord one thousand eight hundred and——— (Seal)

Signed, sealed, and delivered } (Seal)
in presence of } (Seal)